LAST NAME: SOLUTIONS

FIRST NAME: ____________

STUDENT NUMBER: ____________

TEST 1 (A)

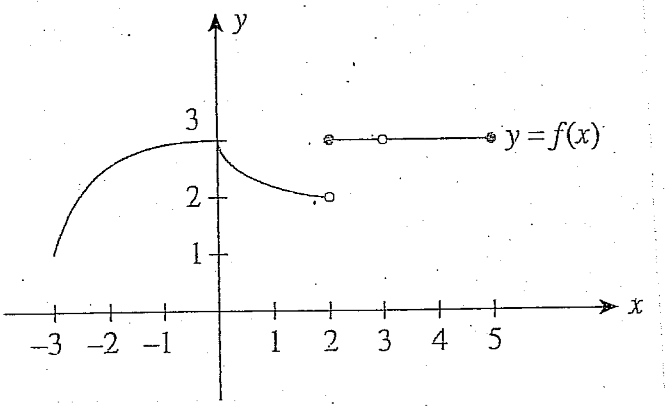
DAWSON COLLEGE

103-DW Section 3 - Calculus 1

Instructor: E. Richer

Date: Sept. 25th 2008

Question 1. *(2 marks each)*



Refer to the graph of the function $f(x)$ to determine whether each statement is true or false. Explain your answer.

(a) $\lim_{x\to -3^+} f(x) = 1$ TRUE FROM GRAPH

(b) $\lim_{x\to 0} f(x) = f(0)$ $f(0) = 3$ $\lim_{x\to 0} f(x) = 3$ TRUE

(c) $\lim_{x\to 2^-} f(x) = 2$ TRUE FROM GRAPH

(d) $\lim_{x\to 2^+} f(x) = 3$ TRUE FROM GRAPH

(e) $\lim_{x\to 3} f(x)$ does not exist. FALSE $\lim_{x\to 3} f(x) = 3$

Question 2.

Evaluate the following limits.

(a) *(2 marks)* $\lim_{x\to 2} \frac{x^2-4}{x^2-5x+6}$

(b) *(2 marks)* $\lim_{x\to 5} \frac{x^3+5x-2}{25-x^2}$

(c) *(3 marks)* $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$

(d) *(3 marks)* $\lim_{x\to\infty} \frac{4x^4-5x^2+x-5}{2x^4+3x^2-7x}$

(a) $\lim_{x\to 2} \frac{x^2-4}{x^2-5x+6} = \lim_{x\to 2} \frac{(x+2)\cancel{(x-2)}}{(x-3)\cancel{(x-2)}} = \lim_{x\to 2} \frac{x+2}{x-3} = \frac{4}{-1} = \boxed{-4}$

(b) $\lim_{x\to 5} \frac{x^3+5x-2}{25-x^2} = \frac{148}{0}$ $\boxed{\text{does Not exist}}$

(c) $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}\left(\frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}\right)$

$= \lim_{x\to 0} \frac{2+x-2}{x(\sqrt{2+x}+\sqrt{2})} = \lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{2+x}+\sqrt{2})} = \lim_{x\to 0} \frac{1}{\sqrt{2+x}+\sqrt{2}}$

$= \frac{1}{\sqrt{2}+\sqrt{2}} = \boxed{\frac{1}{2\sqrt{2}}}$

(d) $\lim_{x\to\infty} \frac{4x^4-5x^2+x-5}{2x^4+3x^2-7x}$

$= \lim_{x\to\infty} \frac{\frac{4x^4}{x^4}-\frac{5x^2}{x^4}+\frac{x}{x^4}-\frac{5}{x^4}}{\frac{2x^4}{x^4}+\frac{3x^2}{x^4}-\frac{7x}{x^4}} = \lim_{x\to\infty} \frac{4-\frac{5}{x^2}+\frac{1}{x^3}-\frac{5}{x^4}}{2+\frac{3}{x^2}-\frac{7}{x^3}}$

$= \frac{4}{2}$

$= \boxed{2}$

Question 3. *(5 marks)*
Determine if the function $g(x)$ is continuous at $x = 2$. Justify your answer using the definition of continuity at a point.

$$g(x) = \begin{cases} 5 & x < 2 \\ 3x - 1 & x = 2 \\ 5 + 2x - x^2 & x > 2 \end{cases}$$

(1) $g(2) = 3(2) - 1 = 5$

so $g(2)$ Exists

(2) $\lim_{x \to 2^-} g(x) = \lim_{x \to 2} 5 = 5$

$\lim_{x \to 2^+} g(x) = \lim_{x \to 2} 5 + 2x - x^2 = 5 + 4 - 4$
$= 5$

So $\lim_{x \to 2} g(x) = 5$ Exists

(3) $\lim_{x \to 2} g(x) = f(2)$

So g is continuous at $x = 2$

Question 4. *(10 marks)*
(a) Use the **limit definition** to find the derivative of $f(x) = 2x^2 - 3x + 2$.
(b) Find the equation of the tangent line to $f(x)$ at the point $(3, 12)$.
(c) Find the point on the graph of $f(x)$ where the tangent line to the curve is horizontal.

(a) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{2(x+h)^2 - 3(x+h) + 2 - [2x^2 - 3x + 2]}{h}$

$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{3x} - 3h + \cancel{2} - \cancel{2x^2} + \cancel{3x} - \cancel{2}}{h}$

$= \lim_{h \to 0} \frac{4xh + 2h^2 - 3h}{h} = \lim_{h \to 0} \frac{h(4x + 2h - 3)}{h}$

$= \lim_{h \to 0} 4x + 2h - 3$

$= 4x - 3$

(b) Slope is $f'(3)$

where $f'(x) = 4x - 3$

$f'(3) = 12 - 3 = 9$

$y = 9x + b$

Substitute $(3, 12)$ to Find b

$12 = 9(3) + b \implies b = 12 - 27 = -15$

The equation is $\boxed{y = 9x - 15}$

Question 5. (*2 marks* each)
Find the derivative of the following functions.
(a) $f(x) = -x^5 + x - 2$
(b) $f(t) = 3t^{\frac{1}{3}} - 6t^{\frac{1}{2}}$
(c) $f(x) = 2x^{-5} + \sqrt{x}$
(d) $f(x) = \frac{3}{x^4} - \frac{3x}{\sqrt{x}}$
(e) $f(t) = \frac{2t^3 - 5t^2 + 7t}{\sqrt[3]{x}}$

(a) $f'(x) = -5x^4 + 1$

(b) $f'(t) = t^{-2/3} - 3t^{-1/2}$

(c) $f(x) = 2x^{-5} + x^{1/2}$

$f'(x) = -10x^{-6} + \frac{1}{2}x^{-1/2}$

(d) $f(x) = 3x^{-4} - 3x^{1/2}$

$f'(x) = -12x^{-5} - \frac{3}{2}x^{-1/2}$

(e) $f(t) = \frac{2t^3}{\sqrt[3]{t}} - \frac{5t^2}{\sqrt[3]{t}} + \frac{7t}{\sqrt[3]{t}}$

$= 2t^{8/3} - 5t^{5/3} + 7t^{2/3}$

$f'(t) = \boxed{\frac{16}{3}t^{5/3} - \frac{15}{3}t^{2/3} + \frac{14}{3}t^{-1/3}}$

Question 6. (*5 marks*)
The height (in m) of a toy rocket t seconds into flight is given by the formula $f(t) = 2t^3 - 15t^2 + 36t$. Find the maximum height that the toy rocket attains.

Velocity $v(t) = f'(t)$
$= 6t^2 - 30t + 36$

MAXIMUM height is AttAINed WHEN
$v(t) = 0$

$6t^2 - 30t + 36 = 0$
$6(t^2 - 5t + 6) = 0$
$6(t-6)(t+1) = 0$
$t = 6$ or $\boxed{\cancel{t = -1}}$
INAdMissAble
time must be positive

SO MAXIMUM height is
$f(6) = 2(6)^3 \; 15(6)^2 - 36(6)$
$= 108$ m