LAST NAME: SOLUTIONS

FIRST NAME: _____

STUDENT NUMBER: _____

TEST 3

DAWSON COLLEGE

103-DW Section 3 - Calculus 1

Instructor: E. Richer

Date: Nov. 13th 2008

Question 1. *(2 marks each)*

Find the derivative of each function.

(a) $f(x) = e^{-2x}\ln(x^2+3x-1)$

(b) $g(t) = \ln(1+\frac{1}{t})$

(c) $f(x) = \frac{\sin x}{\ln x}$

(d) $h(t) = t^2\ln(e^{2t}+1)$

(a) $f'(x) = -2e^{-2x}\ln(x^2+3x-1) + \frac{2x+3}{x^2+3x-1}(e^{-2x})$

(b) $g'(t) = \frac{1}{1+\frac{1}{t}}\left(-\frac{1}{t^2}\right) = \frac{-1}{(t+1)t}$

(c) $f'(x) = \dfrac{\cos x \ln x - \frac{1}{x}\sin x}{(\ln x)^2}$

(d) $h'(t) = 2t\ln(e^{2t}+1) + \frac{1}{e^{2t}+1}e^{2t}\cdot 2\cdot t^2$

Question 2. *(3 marks each)*
Find dy/dx.
(a) $x^2y^2 + y = x - 2$
(b) $\cos(x+y) = 1$
(c) $(x^2+y)^3 = x+y$
(d) $e^{x+y} = xy$

(a) $2xy^2 + 2y\frac{dy}{dx}x^2 + \frac{dy}{dx} = 1$

$(2x^2y+1)\frac{dy}{dx} = 1 - 2xy^2$

$$\boxed{\frac{dy}{dx} = \frac{1-2xy^2}{2x^2y+1}}$$

(b) $-\sin(x+y)\left(1+\frac{dy}{dx}\right) = 0$

$-\sin(x+y) - \sin(x+y)\frac{dy}{dx} = 0$

$$\boxed{\frac{dy}{dx} = \frac{\sin(x+y)}{-\sin(x+y)} = -1}$$

(c) $3(x^2+y)^2\left(2x+\frac{dy}{dx}\right) = 1 + \frac{dy}{dx}$

$6x(x^2+y)^2 + 3(x^2+y)^2\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$$\boxed{\frac{dy}{dx} = \frac{-6x(x^2+y^2)^2+1}{3(x^2+y^2)^2-1}}$$

(d) $e^{x+y}\left(1+\frac{dy}{dx}\right) = y + x\frac{dy}{dx}$ $\quad \boxed{\frac{dy}{dx} = \frac{y-e^{x+y}}{e^{x+y}-x}}$

Question 3. *(5 marks)*

Find the derivative of $y = (x+1)^{(x+1)}$.

$$\ln y = \ln (x+1)^{x+1}$$

$$\ln y = (x+1)\ln(x+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \ln(x+1) + \frac{1}{x+1}(x+1)$$

$$\frac{dy}{dx} = y\left(\ln(x+1)+1\right)$$

$$\frac{dy}{dx} = (x+1)^{x+1}\left(\ln(x+1)+1\right)$$

Question 4. *(5 marks)*

Find $\frac{d^2y}{dx^2}$ of $x^2 - y^2 = 12$, express your answer in terms of x and y only.

$$2x - 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{-2y} = \frac{x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y - x\frac{dy}{dx}}{y^2} = \frac{y - x\left(\frac{x}{y}\right)}{y^2}$$

$$= \frac{y^2 - x^2}{y^3}$$

Question 5. *(5 marks)*

Find the derivative of $f(x) = \ln\left(\frac{(x+1)^2(x-4)^2\cos x}{(2+x)\sin x}\right)$

$$y = \ln\left(\frac{(x+1)^2(x-4)^2\cos x}{(2+x)\sin x}\right)$$ using properties

$$y = 2\ln(x+1) + 2\ln(x-4) + \ln\cos x - \ln(2+x) - \ln\sin$$

$$y' = \frac{2}{x+1} + \frac{2}{x-4} - \frac{\sin y}{\cos y} - \frac{1}{2+x} - \frac{\cos y}{\sin x}$$

Question 6. *(5 marks)*
Find the intervals where the function $f(x) = x^2(3-x)^2$ is increasing and where it is decreasing. Find any relative extrama.

$$f'(x) = 2x(3-x)^2 + 2(3-x)(-1)x^2$$
$$= 2x(3-x)((3-x) - x)$$
$$= 2x(3-x)(3-2x)$$

$f'(x) = 0$ when $x = 0$, $x = 3$, $x = 3/2$

Intervals	$(-\infty, 0)$	$(0, 3/2)$	$(3/2, 3)$	$(3, \infty)$
Test pt	-1	1	2	4
sign of f'	$-$	$+$	$-$	$+$
f incr/ decr	↘	↗	↘	↗

MAX at $x = 3/2$

MAX $(3/2, 81/16)$

$$f(3/2) = (3/2)^2(3 - 3/2)^2 = \frac{81}{16}$$

Min at $x = 0$

Min at $(0, 0)$

$f(0) = 0$

Min at (3)

Min At $(3, 0)$

$f(3) = 0$

Question 7. *(10 marks)*

Find the following information about $f(x) = 2x^3 - 15x^2 + 36x - 20$

The intervals where it is increasing and where it is decreasing

Any relative extrema

The intervals where it is concave up and where it is concave down

Any inflection points

(For 2.5 BONUS MARKS, find the y-intercept and sketch $f(x)$)

$f'(x) = 6x^2 - 30x + 36$

$= 6(x^2 - 5x + 6) = 6(x-2)(x-3)$

$f'(x) = 0$ At $x = 2, x = 3$

Interval	$(-\infty, 2)$	$(2, 3)$	$(3, \infty)$
Test pt.	0	2.5	4
sign of f'	+	−	+
f inc/dec	↗	↘	↗

MAX At $(2, 8)$

Min At $(3, 7)$

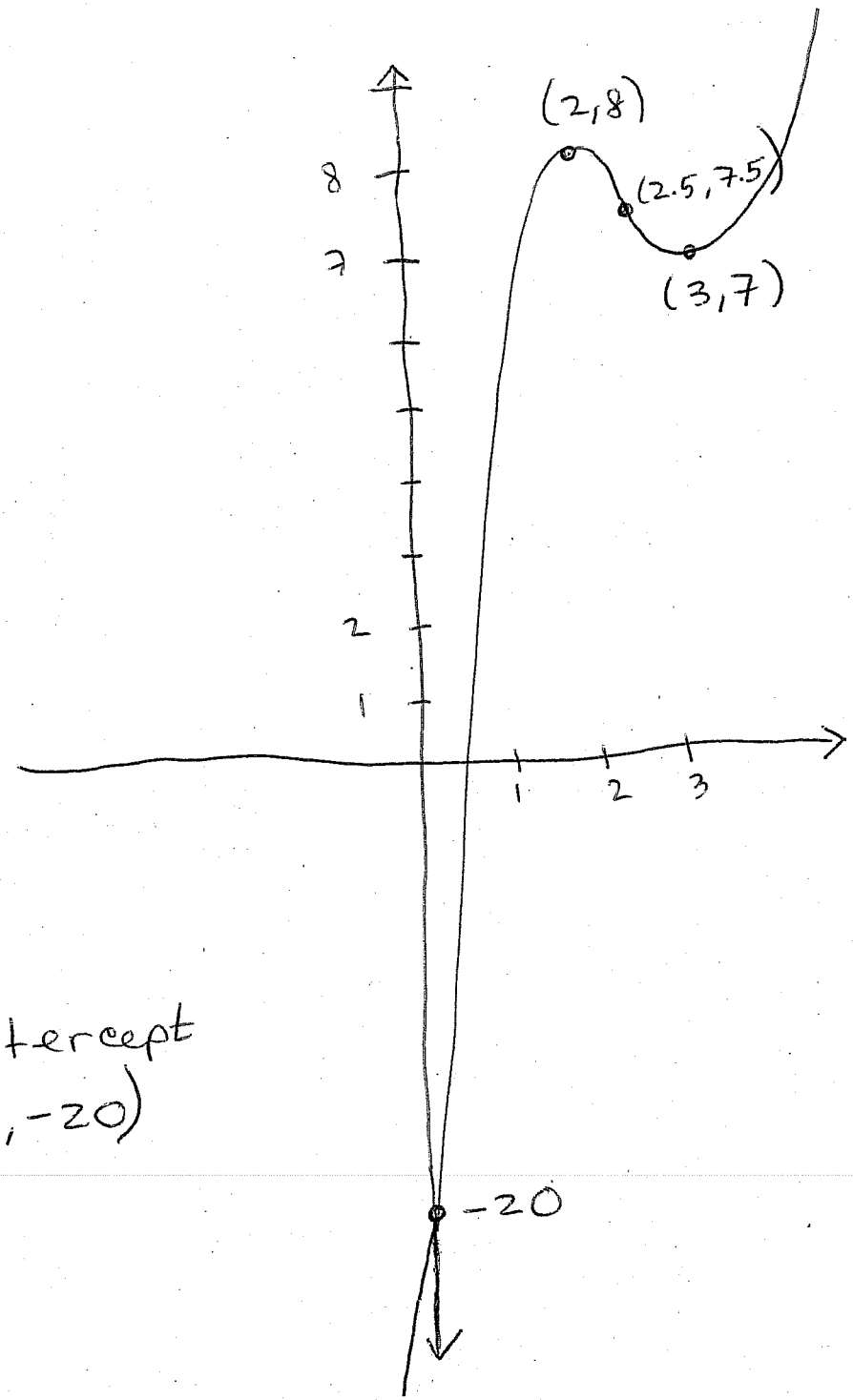
$f''(x) = 12x - 30 = 6(2x - 5)$

$f'' = 0 \Rightarrow$ when $x = 5/2$

Interval	$(-\infty, 5/2)$	$(5/2, \infty)$
test pt	0	3
sign of f''	−	+
concavity	∩	∪

inflection point At $x = 5/2$

$(5/2, 15/2)$



y-intercept
(0,-20)