LAST NAME: SOLUTIONS

FIRST NAME: ____________

STUDENT NUMBER: ____________

TEST 4

DAWSON COLLEGE

103-DW Section 3 - Calculus 1

Instructor: E. Richer

Date: Dec. 5th 2008

Question 1. *(5 marks)*

Find the absolute extrema of $f(x) = (x-1)^2(x-2)$ on the interval [0,2]

$$f'(x) = 2(x-1)(x-2) + (x-1)^2$$
$$= (x-1)[2(x-2) + (x-1)]$$
$$= (x-1)(3x-5)$$

critical #s $x=1$ $x=5/3$

$$f(1) = 0$$
$$f(5/3) = (5/3-1)^2(5/3-2)$$
$$= (2/3)^2(-1/3)$$
$$= -4/27$$
$$f(0) = (0-1)^2(0-2)$$
$$= -2$$
$$f(2) = 0$$

$$\boxed{\begin{array}{ll} \text{MAX} & (2,0)\ \&\ (1,0) \\ \text{MIN} & (0,-2) \end{array}}$$

Question 2.

Consider the function $f(x) = \frac{1}{5}x^5 - 16x$.

(a) *(3 marks)* Find the intervals where f is increasing and decreasing.

(b) *(2 marks)* Find any relative extrema, identify whether they are a maximum or minimum.

(c) *(2 marks)* Find the intervals where f is concave up and concave down.

(d) *(2 marks)* Find any inflection points.

(e) *(5 marks)* Sketch the graph of f.

(a) $f'(x) = x^4 - 16$
$= (x^2-4)(x^2+4)$
$= (x-2)(x+2)(x^2+4)$

critical #s $x = \pm 2$

Interval	$(-\infty,-2)$	$(-2,2)$	$(2,\infty)$
Test pt	-3	0	3
sign of f'	$+$	$-$	$+$
f inc/decr	↗	↘	↗

(b) MAX at $x = -2$
$f(-2) = 25.6$
MAX at $(-2, 25.6)$

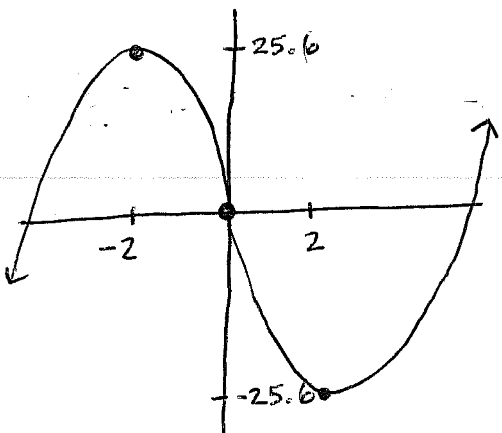
MIN at $x = 2$
$f(2) = -25.6$ Min at $(2, -25.6)$

(c) $f''(x) = 4x^3$
critical # $x = 0$

Interval	$(-\infty, 0)$	$(0,\infty)$
Test pt	-1	1
sign of f''	$-$	$+$
concavity	$\cap$	$\cup$

(d) Inflection point
at $x=0$ $f(0) = 0$ $(0, 0)$

(e)



Question 3. *(6 marks)*
The cost of printing x *Larry Cotter* books is given by $C(x) = 0.2x^2 - 10x + 1280$. The printer has can print a minimum of 5 books and has a maximum capacity of 100 books.
(a) Determine the number of books that should be printed in order to generate the minimum cost.
(b) Determine the number of books that should be printed in order to generate a minimum average cost.

(a) $C'(x) = 0.4x - 10$

$x = \frac{10}{0.4} = 25 \qquad [5, 100]$

$C(25) = 0.2(25)^2 - 10(25) + 1280 = 1155$
$C(5) = 0.2(5)^2 - 10(5) + 1280 = 1235$
$C(100) = 0.2(100)^2 - 10(100) + 1280 = 2280$

minimum cost of 1155 \$ when $\boxed{x = 25}$

(b) $\bar{C}(x) = 0.2x - 10 + \frac{1280}{x}$

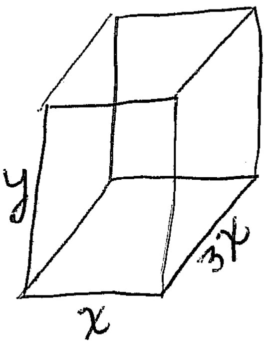
$\bar{C}'(x) = 0.2 - \frac{1280}{x^2} \qquad 0.2x^2 = 1280$
$x = \pm 80$

$x = 0$ is also a critical number
only $x = 80$ in the interval $[5, 100]$

$\bar{C}(80) = 0.2(80) - 10 + \frac{1280}{80} = 22$
$\bar{C}(5) = 0.2(5) - 10 + \frac{1280}{5} = 247$
$\bar{C}(100) = 0.2(100) - 10 + \frac{1280}{100} = 22.8$

minimum average cost when $\boxed{x = 80}$.

Question 4. *(6 marks)*
You are to build a rectangular box out of cardboard. It must have a volume of $144 cm^3$ and its base must be three times its width. Determine the dimensions of the box so that a minimum amount of cardboard is used?



SURFACE AreA;

$$A = x(3x) + 2xy + 2(3x)y$$
$$= 3x^2 + 2xy + 6xy$$
$$= 3x^2 + 8xy$$

$V = 144$ $\qquad 144 = x(3x)y$

$$y = \frac{144}{3x^2}$$

$$A = 3x^2 + 8x\left(\frac{144}{3x^2}\right)$$

$$A = 3x^2 + \frac{384}{x}$$

$$A' = 6x - \frac{384}{x^2}$$
$$= \frac{6x^3 - 384}{x^2} = \frac{6(x^3 - 64)}{x^2}$$

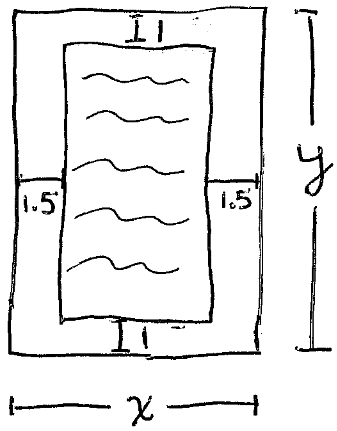
critical #s $\quad x = 0$ & $x = 4$

only $x = 4$ is possible

DiMENSiONS $\quad x = 4$
$3x = 12$
$y = \frac{144}{3(4)^2} = 3$

DIMENSIONS ARE $\boxed{3CM \times 4CM \times 12CM}$

Question 5. *(6 marks)*
A poster is to be printed with 1 inch margins on the top and bottom and 1.5 inch margins on each side. The poster must have an area of 73.5 square inches. Determine the dimensions of the poster in order to maximize the printed area.



Area of printed area:

$$A_p = (x-3)(y-2)$$

Area of poster:

$$xy = 73.5$$

$$y = \frac{73.5}{x}$$

$$A_p = (x-3)\left(\frac{73.5}{x} - 2\right) \qquad \text{Bounds } [3, 36.75]$$

Expand:

$$A_p = 73.5 - \frac{220.5}{x} - 2x + 6$$

$$A_p' = \frac{220.5}{x^2} - 2$$

critical #s

$$\frac{220.5 - 2x^2}{x^2} = 0$$

$$x^2 = \frac{-220.5}{-2} = 110.25$$

$$x = \pm 10.5 \quad \& \quad x = 0$$

only $x = 10.5$ is in the interval

$A_p(3) = 0$, $A_p(36.75) = 0$, $A_p(10.5) = 7.5(5) = 37.5$

Maximum printed area is 37.5 in^2.

$$\boxed{x = 10.5 \quad y = 7}$$

Question 6. *(5 marks)*
Find the integral.
(a) $\int \frac{1}{x^4} - \frac{1}{\pi^3} dx$
(b) $\int x^3 - 2x^2 + \frac{\sqrt{x}}{2} - \frac{1}{x^2} dx$

(a) $\int x^{-4} - \frac{1}{\pi^3}\, dx = \frac{x^{-3}}{-3} - \frac{1}{\pi^3} x + C$

$= \boxed{-\frac{1}{3x^3} - \frac{1}{\pi^3} x + C}$

(b) $\int x^3 - 2x^2 + \frac{1}{2}x^{1/2} - x^{-2}\, dx$

$= \frac{x^4}{4} - \frac{2x^3}{3} + \frac{1}{2}\frac{x^{3/2}}{3/2} - \frac{x^{-1}}{-1} + C$

$= \boxed{\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^{3/2}}{3} + \frac{1}{x} + C}$

Question 6. *(5 marks)*
Find the integral.
(a) $\int \sin x - e^x + \frac{\pi^3}{x} dx$

(b) $\int x^3 - 2x^2 + \frac{\sqrt{x}}{2} - \frac{1}{x^2} dx$

(a) $-\cos x - e^x + \pi^3 \ln x + C$

(b) $\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^{3/2}}{3} + \frac{1}{x} + C$

Question 7. *(12 marks)*
Find the integral.
(a) $\int x\sqrt{3x-2}dx$

(b) $\int \frac{4x^3-x+\sqrt{x}+2}{\sqrt{x}}dx$

(c) $\int (3x^2-x)(1-x^2)dx$

(d) $\int \frac{x^2}{(x^3-1)^4}dx$

(a) $U = 3x-2$ (this one is not solved ~~how~~ using the regular method of substitution sorry)

$du = 3dx$

$\int x\sqrt{u}\,\frac{du}{3}$ but $u = 3x-2$

$u+2 = 3x$

$x = \frac{u+2}{3}$

$\int x\sqrt{u}\,\frac{du}{3} = \int \frac{u+2}{3}\sqrt{u}\,\frac{du}{3} = \int \left(\frac{u+2}{9}\right)\sqrt{u}\,du$

$= \frac{1}{9}\int u^{3/2} + 2u^{1/2}\,du$

$= \frac{1}{9}\ \frac{u^{5/2}}{5/2} + 2\frac{u^{3/2}}{3/2} + C$

$= \boxed{\frac{2}{45}(3x-2)^{5/2} + \frac{4}{3}(3x-2)^{3/2} + C}$

(b) $\int 4x^{5/2} - x^{1/2} + 1 + 2x^{-1/2}\,dx$

$= 4\frac{x^{7/2}}{7/2} - \frac{x^{3/2}}{3/2} + x + 2\frac{x^{1/2}}{1/2} + C$

$\boxed{= \frac{8}{7}x^{7/2} - \frac{2}{3}x^{3/2} + x + 4x^{1/2} + C}$

(c) $\int 3x^2 - x - 3x^4 + x^3\,dx$

$= \boxed{x^3 - x^2/2 - 3\frac{x^5}{5} + \frac{x^4}{4} + C}$

(d) $U = x^3-1$ $\quad du = 3x^2dx$

$dx = \frac{du}{3x^2}$

$\int \frac{\cancel{x^2}}{u^4}\frac{du}{3\cancel{x^2}} = \int \frac{1}{3}u^{-4}du$

$= \frac{1}{3}\frac{u^{-3}}{-3} + C$

$\boxed{= -\frac{1}{9}(x^3-1)^{-3} + C}$