PRACTISE EXERCISES FOR TEST 3 (SOLUTIONS)-MATH 103DW

3.6 (Implicit Differentiation)

5.5 (Differentiation Involving Logarithms)

4.1 (Applications of 1st Derivative)

4.2 (Applications of 2nd Derivative)

4.3 (Curve Sketching) BONUS

(1) Find dy/dx

a- $\frac{2x}{y}$

b- $\frac{-y^2}{2xy+1}$

c- $\frac{-y}{x}$

d- $\frac{1-e^{x+y}}{e^{x+y}}$

e- $\frac{-2xy\ln y}{x^2-y}$

f- $\frac{4x^{\frac{1}{2}}y-1}{-2x^{\frac{3}{2}}}$

g- $\frac{y-3x^2y}{x^3-x}$

h- $\frac{1+y\sin(x+y)}{\cos(x+y)-y\sin(x+y)}$

i- $\frac{x^2y-2y}{2xy}$

j- $\frac{1-2xe^y}{x^2e^y-1}$

(2) Find the equation of the tangent line to the curve at the given point.

a- $y=2x-1$

b- $y=-6x+8$

c- $y=\frac{11}{3}x+4$

d- $y=x-1$

e- derivative does not exist at this point

(3) Find dy^2/d^2x (your answer should be in terms of x and y only)

a- $\frac{-y^2-x^2}{y^4}$

b- $\frac{12xy^2-9x^4}{2y^3}$

c- $\frac{2y}{(1-x)^2}$

d- $\frac{-4}{y^3}$

(4) Find the derivative (your answer should be in terms of x only)

a- $\frac{2x}{x^2+1}$

b- $\frac{1}{x\ln x}$

c- $\frac{-2\sin x}{\cos x}$

d- $\frac{x(x-1)^{\frac{3}{2}}}{\sqrt{x+1}}(\frac{1}{x}+\frac{3}{2(x-1)}-\frac{1}{2(x+1)})$

e- $\frac{x}{x^2+1}(\frac{1}{x}-\frac{2x}{x^2+1})$

f- $2x(\ln x^2+1)$

g- $2\cos 2x\ln x^2+\frac{2}{x}\sin 2x$

h- $(x+1)^x(\ln(x+1)+\frac{x}{x+1})$

i- $x^{\cos x}(-\sin x(\ln x)+\frac{\cos x}{x})$

j- $(\ln x)^x(\ln(\ln x)+\frac{1}{\ln x})$

k- $\ln x+1$

(5) Find the points on the curve of the function where the tangent line is horizontal.

a- $(-2,0),(4,0),(1,\frac{-81}{8})$

b- $(1,\frac{1}{2}),(-1,\frac{1}{2})$

(6) Find all relative extrema of the function.

a- Maximum at $(\sqrt{6},\frac{\sqrt{6}+3}{6})$, Minimum at $(-\sqrt{6},\frac{-\sqrt{6}+3}{6})$

b- Maximum at $(-2,20)$, Minimum at $(1,-7)$

c- Note that the domain is $-4\leq x\leq 4$, Maximum at $(\sqrt{8},16\sqrt{2})$ and Minimum at $(-\sqrt{8},-16\sqrt{2})$

d- Maximum at $(-1,\frac{4}{5})$, Minimum at $(1,-\frac{4}{5})$

(7) Find the relative extrema, the intervals where the function is increasing/decreasing, the inflection points, the intervals where the function is concave up/down. (Bonus, sketch the graph indicating intercepts as well)

a- Minimums at $(0,0)$ and $(6,0)$ and maximum at $(3,81)$. Inflection points at $(3-\sqrt{3},36)$ and $(3+\sqrt{3},36)$.

b- Minimum at $(6,-81)$ and maximum at $(0,27)$. Inflection point at $(3,-27)$.

c- Minimum at $(3,-27)$. Inflection points at $(0,0)$ and $(2,-16)$

d- No relative extrema. Inflection point at $(2,8)$

e- Minimum at $(1,-3)$ and no inflection points (always concave up).

f- No relative extrema. Inflection point at $(1,0)$.

g- Minimum at $(0,-4)$. No inflection points (always concave down).

h- Minimum at $(-2,-4)$ and $(2,-4)$, maximum at $(0,0)$.

Inflection points $(-\sqrt{\frac{4}{3}},-\frac{20}{9})$ and $(\sqrt{\frac{4}{3}},-\frac{20}{9})$

i- Minimum at $(3,-25)$ and inflection point $(0,2)$

j- Maximum at $(5,0)$, no inflection points (concave down everywhere).