Review Exercises - Test 4
MATH103

* For SECTION 4.3 ; Graph sketching.
REFER TO TEST 3 review Exercises

SECTIONS 4.4 & 4.5

(1) Find the olute extrema on the given interval

(a) $f(x) = x^{10} - 10x$ $[0,2]$

(b) $f(x) = x - \ln x$ $[0.1, 2]$

(c) $f(x) = \frac{1}{3}x^3 - x^2 + x + 1$ $[0,2]$

(d) $h(t) = t^3 - 6t^2$ $[2,5]$

(e) $g(x) = \frac{x}{x^2+1}$ $[0,5]$

(f) $f(x) = (x-2)^3$ $[-1,4]$

(g) $f(x) = x\sqrt{1-x^2}$ $[-1,1]$

(h) $g(x) = 2x^3 - 6x^2 + 6x + 1$ $[-2,0]$

(i) $f(x) = x + 9/x$ $[2,5]$

(j) $g(x) = x^4 - 2x^2$ $[-2,2]$

(2) The profit of a company is given by $P(x) = -x^2 + 8x + 20$ where x is the amount of money spent on advertising (in thousands of $).

What should be the advertising budget in order to maximize profits?

(3) A rectangular page is to contain 24 square cm of print. The margins at the top & bottom are 1½ cm & the side margins are 1cm each. What should the dimensions of the page be to minimize the area of the page?

(4) Determine the dimensions of a rectangular solid with square base if it is to have a maximum volume & a surface area of 337.5 cm^2.

(5) The weekly demand for DVD's is given by

$$p = -0.0005x^2 + 60$$

where p denotes the unit price in dollars & x denotes the quantity demanded. The weekly total cost function associated with producing the DVDs is given by

$$C(x) = -0.001x^2 + 18x + 4000$$

where $C(x)$ denotes the total cost to produce x dvds.

Find the production level that will maximize profits. (Hint: use quadratic formula)

(6) Monthly profits for the sale of x cameras is given by $P(x) = -0.04x^2 + 240x - 10000$ (where $P(x)$ is in $). How many cameras should be sold to maximize profits?

(7) Total cost to manufacture x units is given by $C(x) = 0.001x^2 + 100x + 4000$

(a) Find average cost function $\overline{C}(x)$

(b) Determine the production level that will minimize average cost

(8) A 3000 squre foot garden is to be enclosed. Material along 3 sides of the garden costs 25$/foot & 10$/foot along the 4th side. What is the minimum cost to enclose the fence?

(9) If you have 100 ft of fencing & want to enclose a rectangular area up against a straight wall that does not require fencing. What is the largest area you can enclose.

(10) A rectangular swimming pool is to be built with an area of 1800 square ft. The owner wants 5 ft wide decks along two sides of the pool & 10 ft wide decks at the two ends. Find dimensions of the smallest piece of property on which the pool can be built.

(11) Find the integral

(a) $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx$

(b) $\int (4x^3 + 6x^2 - 1)\, dx$

(c) $\int \frac{1}{x\sqrt{x}}\, dx$

(d) $\int x(x^2+3)\, dx$

(e) $\int \frac{x^2 + x + 1}{\sqrt{x}}\, dx$

(f) $\int \frac{x^2 + 2x - 3}{x^4}\, dx$

(g) $\int (2t^2 - 1)^2\, dt$

(h) $\int t^3 \sqrt{t^4 + 5}\, dt$

(i) $\int x^3 (x^4 + 3)^2\, dx$

(j) $\int \sqrt[3]{(1 - 2x^2)}\, (-4x)\, dx$

(k) $\int \frac{x^2 - 1}{x^{3/2}}\, dx$

(l) $\int 2\sin x + 3\cos x\, dx$

(m) $\int t^2 - \sin t\, dt$

(n) $\int e^x + \frac{3}{x} - 5\cos x\, dx$

(o) $\int t^2 \left(t - \frac{2}{t}\right) dt$

(p) $\int \frac{x^2}{(16 - x^3)^2}\, dx$

(q) $\int 5x \sqrt[3]{1 - x^2}\, dx$

ANSWERS

1. @ MAX (2, 1004)
   MIN (1, −9)

   (b) MAX (2, 1.3)
   MIN (0.1, −2.2)

   (c) MAX $(2, 253\frac{1}{3})$
   MIN (0, 1)

   (d) MAX (2, −16)
   MIN (4, −32)

   (e) MAX $(1, \frac{1}{2})$
   MIN (0, 0)

   (f) MAX (−1, 27)
   MIN (2, 0)

   (g) MAX $(\frac{1}{\sqrt{2}}, \frac{1}{2})$
   MIN $(-\frac{1}{\sqrt{2}}, -\frac{1}{2})$

   (h) MAX (−2, 54)
   MIN (1, 0)

   (i) MAX (5, 6.8)
   MIN (3, 6)

   (j) MAX (−2, 8) & (2, 8)
   MIN (−1, −1) & (1, −1)

2. 4000 $

3. 9cm x 6cm

4. 5.3cm x 14.6cm

(5) 168 DVDs

(6) 3000 CAMERAS

(7) (a) $\bar{C}(x) = 0.001x + 100 + 4000x^{-1}$

(b) $x = 2000$

(8) $2\sqrt{10}$ ft X $150\sqrt{10}$ ft

(9) 1250 square feet

(10) 60 ft X 30 ft

(11) (a) $\frac{2}{3}x^{3/2} + \sqrt{x} + C$

(b) $x^4 + 2x^3 - x + C$

(c) $-\frac{2}{\sqrt{x}} + C$

(d) $\frac{x^4}{4} + 3\frac{x^2}{2} + C$

(e) $\frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} - 2\sqrt{x} + C$

(f) $-\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C$

(g) $\frac{4t^5}{5} - \frac{4t^3}{3} + t + C$

(h) $\frac{1}{6}(t^4+5)^{3/2} + C$

(i) $\frac{1}{12}(x^4+3)^3 + C$

(j) $\frac{3}{4}(1-2x^2)^{4/3} + C$

(k) $\frac{2}{3}x^{3/2} + \frac{2}{\sqrt{x}} + C$

(l) $2\cos x - 3\sin x + C$

(m) $t^3/3 + \cos t + C$

(n) $e^x + 3\ln x - 5\sin x + C$

(o) $\frac{t^4}{4} - t^2 + C$

(p) $-\frac{1}{3}(16-x^3)^{-1} + C$

(q) $-\frac{15}{8}(1-x^2)^{4/3} + C$