

①

REVIEW EXERCISES - TEST 4
MATH103

* FOR SECTION 4.3 ; Graph sketching.
REFER TO TEST 3 review Exercises

SECTIONS 4.4 & 4.5

① Find the Absolute extrema on the given interval

(a) $f(x) = x^{10} - 10x$ $[0, 2]$

(b) $f(x) = x - \ln x$ $[0.1, 2]$

(c) $f(x) = \frac{1}{3}x^3 - x^2 + x + 1$ $[0, 2]$

(d) $h(t) = t^3 - 6t^2$ $[2, 5]$

(e) $g(x) = \frac{x}{x^2 + 1}$ $[0, 5]$

(f) $f(x) = (x - 2)^3$ $[-1, 4]$

(g) $f(x) = x\sqrt{1 - x^2}$ $[-1, 1]$

(h) $g(x) = 2x^3 - 6x^2 + 6x + 1$ $[-2, 0]$

(i) $f(x) = x + 9/x$ $[2, 5]$

(j) $g(x) = x^4 - 2x^2$ $[-2, 2]$

- ② THE PROFIT OF A COMPANY IS GIVEN BY $P(x) = -x^2 + 8x + 20$ WHERE x IS THE AMOUNT OF MONEY SPENT ON ADVERTISING (IN THOUSANDS OF \$).

WHAT SHOULD BE THE ADVERTISING BUDGET IN ORDER TO MAXIMIZE PROFITS?

- ③ A RECTANGULAR PAGE IS TO CONTAIN 24 SQUARE CM OF PRINT. THE MARGINS AT THE TOP & BOTTOM ARE $1\frac{1}{2}$ CM & THE SIDE MARGINS ARE 1 CM EACH. WHAT SHOULD THE DIMENSIONS OF THE PAGE BE TO MINIMIZE THE AREA OF THE PAGE?

- ④ DETERMINE THE DIMENSIONS OF A RECTANGULAR SOLID WITH SQUARE BASE IF IT IS TO HAVE A MAXIMUM VOLUME & A SURFACE AREA OF 337.5 cm^2 .

- ⑤ THE WEEKLY DEMAND FOR DVD'S IS GIVEN BY $p = -0.0005x^2 + 60$

WHERE p DENOTES THE UNIT PRICE IN DOLLARS & x DENOTES THE QUANTITY DEMANDED. THE WEEKLY TOTAL COST FUNCTION ASSOCIATED WITH PRODUCING THE DVD'S IS GIVEN BY

$$C(x) = -0.001x^2 + 18x + 4000$$

WHERE $C(x)$ DENOTES THE TOTAL COST TO PRODUCE x DVD'S.

FIND THE PRODUCTION LEVEL THAT WILL MAXIMIZE PROFITS. (HINT: USE QUADRATIC FORMULA)

- ⑥ MONTHLY PROFITS FOR THE SALE OF x CAMERAS IS GIVEN BY $P(x) = -0.04x^2 + 240x - 10000$ (WHERE $P(x)$ IS IN \$). HOW MANY CAMERAS SHOULD BE SOLD TO MAXIMIZE PROFITS?

⑦ Total cost to MANUFACTURE x UNITS is given by $C(x) = 0.001x^2 + 100x + 4000$

(a) Find AVERAGE cost function $\bar{C}(x)$

(b) Determine the production level that will minimize AVERAGE cost

⑧ A 3000 square foot garden is to be enclosed. Material along 3 sides of the garden costs 25\$/foot & 10\$/foot along the 4th side. What is the minimum cost to enclose the fence? (Garden is rectangle)

⑨ If you have 100 ft of fencing & want to enclose a rectangular area up against a straight wall that does not require fencing. What is the largest area you can enclose.

⑩ A rectangular swimming pool is to be built with an area of 1800 square ft. The owner wants 5 ft wide decks along two sides of the pool & 10 ft wide decks at the two ends. Find dimensions of the smallest piece of property on which the pool can be built.

⑪ Find the INTEGRAL

(a) $\int (\sqrt{x} + \frac{1}{2\sqrt{x}}) dx$

(b) $\int (4x^3 + 6x^2 - 1) dx$

(c) $\int \frac{1}{x\sqrt{x}} dx$

(d) $\int x(x^2 + 3) dx$

(4)

$$(e) \int \frac{x^2 + x + 1}{\sqrt{x}} dx$$

$$(f) \int \frac{x^2 + 2x - 3}{x^4} dx$$

$$(g) \int (2t^2 - 1)^2 dt$$

$$(h) \int t^3 \sqrt{t^4 + 5} dt$$

$$(i) \int x^3 (x^4 + 3)^2 dx$$

$$(j) \int \sqrt[3]{(1 - 2x^2)} (-4x) dx$$

$$(k) \int \frac{x^2 - 1}{x^{3/2}} dx$$

$$(l) \int 2\sin x + 3\cos x dx$$

$$(m) \int t^2 - \sin t dt$$

$$(n) \int e^x + \frac{3}{x} - 5\cos x dx$$

$$(o) \int t^2 \left(t - \frac{2}{t} \right) dt$$

$$(p) \int \frac{x^2}{(16 - x^3)^2} dx$$

$$(q) \int 5x \sqrt[3]{1 - x^2} dx$$

ANSWERS

①

a) MAX (2, 1004)
MIN (1, -9)

b) MAX (0.1, 2.4)
MIN (1, 1)

c) MAX (2, 5/3)
MIN (0, 1)

d) MAX (2, -16)
MIN (4, -32)

e) MAX (1, 1/2)
MIN (0, 0)

f) MAX (4, 8)
MIN (-1, -27)

g) MAX (1/√2, 1/2)
MIN (-1/√2, -1/2)

h) MAX (1, 3)
MIN (-2, -51)

i) MAX (5, 6.8)
MIN (3, 6)

j) MAX (-2, 8) & (2, 8)
MIN (-1, -1) & (1, -1)

② 4000 \$

③ 9cm x 6cm

④ 5.3cm x 14.6cm Open Box
7.5cm x 7.5cm Closed box

5) 168 DVDs

6) 3000 CAMERAS

7) (a) $\bar{C}(x) = 0.001x + 100 + 4000x^{-1}$

(b) $x = 2000$

8) $10\sqrt{21}$ ft \times $\frac{300}{\sqrt{21}}$ ft

9) 1250 square feet

10) 60 ft \times 30 ft

11) (a) $\frac{2}{3}x^{3/2} + \sqrt{x} + C$

(b) $x^4 + 2x^3 - x + C$

(c) $-\frac{2}{\sqrt{x}} + C$

(d) $\frac{x^4}{4} + 3\frac{x^2}{2} + C$

(e) $\frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + 2\sqrt{x} + C$

(f) $-\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + C$

(g) $\frac{4t^5}{5} - \frac{4t^3}{3} + t + C$

(n) $e^x + 3\ln x - 5\sin x + C$

(h) $\frac{1}{6}(t^4 + 5)^{3/2} + C$

(o) $\frac{t^4}{4} - t^2 + C$

(i) $\frac{1}{12}(x^4 + 3)^3 + C$

(p) $\frac{1}{3}(16 - x^3)^{-1} + C$

(j) $\frac{3}{4}(1 - 2x^2)^{4/3} + C$

(q) $-\frac{15}{8}(1 - x^2)^{4/3} + C$

(k) $\frac{2}{3}x^{3/2} + \frac{2}{\sqrt{x}} + C$

(l) $-2\cos x + 3\sin x + C$

(m) $\frac{t^3}{3} + \cos t + C$