SOLUTIONS TEST 2
103 DW-04

1- (a) $f'(x) = -e^{-x}\cos x - \sin y\,(e^{-x})$

(b) $g'(t) = \left(6t^2 - \frac{1}{2}t^{-1/2}\right)\tan t + \sec^2 t\,(2t^3 - \sqrt{t})$

(c) $f'(x) = \dfrac{2x(x\sin x) - (\sin x + x\cos x)(x^2-1)}{x^2\sin^2 x}$

(d) $\dfrac{(8t^3 + t^{-2})(t+2) + (2t^4 - t^{-1})}{(t+2)^2} - \sin t$

2- (a) $g'(t) = 3(\sin t\,(t^3 - 2t))^2\left(\cos t\,(t^3-2t) + (3t^2-2)\sin t\right)$

(b) $f'(t) = -4(\cos^{-5}(5t))(-\sin(5t))\cdot 5$

(c) $h'(x) = \frac{1}{5}\left(\dfrac{\sin x}{x^5 - 2x^2}\right)^{-4/5}\left(\dfrac{\cos x(x^5-2x^2) - (5x^4-4x)\sin x}{(x^5-2x^2)^2}\right)$

(d) $f'(x) = \cos\left(e^{\cos 2x}\right)\cdot e^{\cos 2x}\cdot -\sin(2x)\cdot 2$

3- slope is derivative

$$y' = e^{2x-3}\cdot 2$$

At $x = 3/2$ $\quad y' = e^{2(\frac{3}{2})-3}\cdot 2$

$= e^0 \cdot 2 = 2$

So $y = 2x + b$ find b $\quad 1 = 2(\frac{3}{2}) + b$

$1 = 3 + b$

$\boxed{y = 2x - 2}$

4- $f'(x) = 2(3x-2)\cdot 3(2x+2)^3 + 3(2x+2)^2 \cdot 2(3x-2)^2$

$= 6(3x-2)(2x+2)^3 + 6(2x+2)^2(3x-2)^2$

$= 6(3x-2)(2x+2)^2\left[(2x+2) + (3x-2)\right]$

$= 6(3x-2)(2x+2)^2(5x)$

$= 30x(3x-2)(2x+2)^2$

$= 30(3x^2-2x)(2x+2)^2$

$f''(x) = 30\left[(6x-2)(2x+2)^2 + 2(2x+2)\cdot 2(3x^2-2x)\right]$

5- $f(x) = \dfrac{2x^3 - 4x + x^{-2}}{x^{1/3}}$

$= 2x^{8/3} - 4x^{2/3} + x^{-7/3}$

$f'(x) = \frac{16}{3}x^{5/3} - \frac{8}{3}x^{-1/3} - \frac{7}{3}x^{-10/3}$

$f''(x) = \frac{80}{3^2}x^{2/3} + \frac{8}{3^2}x^{-4/3} + \frac{70}{3^2}x^{-13/3}$

6- (a) $R(x) = x \cdot p$

$= x(900 - 0.75x)$

$= 900x - 0.75x^2$

$P(x) = R(x) - C(x)$

$= -0.000003x^3 - 0.045x^2 + 300x - 120000$

(b) $P'(x) = -0.000009x^2 - 0.09x + 300$

(c) $P(1000) = 252300\ \$$

yes there is a profit

(d) $P'(1000) = 2901$

yes they should increase sales because $P'(1000)$ is positive meaning profit is increasing as x increases)

(e) $\bar{C}(x) = \frac{C(x)}{x} = 0.000003x^2 - 0.03x + 600 + \frac{120000}{x}$

$\bar{C}'(x) = 0.000006x - 0.03 - \frac{120000}{x^2}$

$\bar{C}'(1000) = -0.09$

the average cost is decreasing because $C'(1000)$ is negative

BONUS

$f(x) = e^{-4x}$

$f'(x) = -4e^{-4x}$

$f''(x) = 16e^{-4x} = 4^2 e^{-4x}$

$f'''(x) = -4^3 e^{-4x}$

$f^4(x) = 4^4 e^{-4x}$

$f^5(x) = -4^5 e^{-4x}$

$\vdots$

$f^{67}(x)$ is odd so sign is negative

$f^{67}(x) = -4^{67} e^{-4x}$.