LAST NAME: Solutions

FIRST NAME: ____________

STUDENT NUMBER: ____________

TEST 3

DAWSON COLLEGE

103-DW Section 4 - Calculus 1

Instructor: E. Richer

Date: Nov. 14th 2008

Question 1. *(2 marks each)*

Find the derivative of each function.

(a) $f(x) = \sin(2x)\ln(x^2+3x-1)$

(b) $g(t) = \ln(t+\frac{1}{t})$

(c) $f(x) = \frac{e^{-2x}+1}{\ln x}$

(d) $h(t) = (\ln(t^2+1))^3$

(a) $2\cos 2x \ln(x^2+3x-1) + \frac{2x+3}{x^2+3x-1}\sin 2x$

(b) $g' t = \frac{1}{t+\frac{1}{t}}\left(1-\frac{1}{t^2}\right)$

$= \frac{t}{t^2+1}\left(\frac{t^2-1}{t^2}\right) = \frac{t^2-1}{t(t^2+1)}$

(c) $f'(x) = \frac{-2e^{-2x}\ln x - \frac{1}{x}(e^{-2x}+1)}{(\ln x)^2}$

(d) $h'(t) = 3(\ln(t^2+1))^2\left(\frac{2t}{t^2+1}\right)$

Question 2. *(3 marks each)*
Find dy/dx.
(a) $\sin(xy) = 2x + 5$
(b) $x \ln y + y^3 = \ln x$
(c) $x^2 + xy - y^3 = xy^2$

(a) $\cos(xy)\left(y + x\frac{dy}{dx}\right) = 2$

$$\boxed{\frac{dy}{dx} = \frac{2 - y\cos(xy)}{x\cos(xy)}}$$

(b) $\ln y + \frac{x}{y}\frac{dy}{dx} + 3y^2\frac{dy}{dx} = \frac{1}{x}$

$$\frac{dy}{dx}\left(\frac{x}{y} + 3y^2\right) = \frac{1}{x} - \ln y$$

$$\boxed{\frac{dy}{dx} = \frac{\frac{1}{x} - \ln y}{\frac{x}{y} + 3y^2}} = \frac{(x - \ln y)(y)}{(x + 3y^3)(x)}$$

(c) $2x + y + x\frac{dy}{dx} - 3y^2\frac{dy}{dx} = y^2 + 2yx\frac{dy}{dx}$

$$\boxed{\frac{dy}{dy} = \frac{y^2 - 2x - y}{x - 3y^2 - 2xy}}$$

Question 3. *(3 marks)*
Find the equation of the tangent line to the curve $xy^2 = 1$ at the point $(1, -1)$.

$$y^2 + 2y\frac{dy}{dx}x = 0$$

$$\frac{dy}{dx} = \frac{-y^2}{2xy} \quad \text{At } (1,-1) \qquad \frac{-(-1)^2}{2(1)(-1)} = \frac{-1}{-2} = \frac{1}{2}$$

$$y = \frac{1}{2}x + b$$

$$-1 = \frac{1}{2} + b \qquad b = -3/2$$

$$\boxed{y = -\tfrac{1}{2}x - 3/2}$$

Question 4. *(5 marks)*
Find the derivative of $y = (x+1)^{\ln x}$.

$$\ln y = \ln (x+1)^{\ln x}$$

$$\ln y = \ln x \ln(x+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\ln(x+1) + \frac{1}{x+1}\ln y$$

$$\boxed{\frac{dy}{dx} = (x+1)^{\ln x}\left(\frac{\ln(x+1)}{x} + \frac{\ln x}{x+1}\right)}$$

Question 5. *(5 marks)*
Given $xy - 1 = y - x$, find $\frac{d^2y}{dx^2}$, express your answer in terms of x and y only.

$$y + x\frac{dy}{dx} = \frac{dy}{dx} - 1$$

$$\frac{dy}{dx} = \frac{-1-y}{x-1}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{dy}{dx}(x-1) - (-1-y)}{(x-1)^2}$$

$$= \frac{-\left(\frac{-1-y}{\cancel{x-1}}\right)\cancel{(x-1)} + 1 + y}{(x-1)^2}$$

$$= \frac{1+y+1+y}{(x-1)^2} = \boxed{\frac{2(y+1)}{(x-1)^2}}$$

Question 6. *(5 marks)*

Find the derivative of $y = \ln\left(\frac{x\sqrt{x-1}}{(x+2)^{\frac{3}{2}}\sin^2 x}\right)$

$$y = \ln x + \frac{1}{2}\ln(x-1) - \left(\frac{3}{2}\ln(x+2) + 2\ln\sin x\right)$$

$$\boxed{\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2(x-1)} - \frac{3}{2(x+2)} + \frac{2\cos x}{\sin x}}$$

Question 7. *(5 marks)*
Find the intervals where the function $f(x) = (x+1)^2(x-2)^2$ is increasing and where it is decreasing. Find all relative extrema.

$$f'(x) = 2(x+1)(x-2)^2 + 2(x-2)(x+1)^2$$
$$= 2(x+1)(x-2)(x-2+x+1)$$
$$= 2(x+1)(x-2)(2x-1)$$

Critical Numbers $x=-1$ $x=2$ $x=\frac{1}{2}$

Interval	$(-\infty,-1)$	$(-1,\frac{1}{2})$	$(\frac{1}{2},2)$	$(2,\infty)$
Test pt	-2	0	1	3
sign of f'	$-$	$+$	$-$	$+$
f incr/decr	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$

$\boxed{\text{MAX At } x=\frac{1}{2} \quad (\frac{1}{2}, \frac{81}{16})}$ $f(\frac{1}{2}) = (\frac{3}{2})^2(-\frac{3}{2})^2 = \frac{81}{16}$

$\boxed{\text{MIN At } x=-1 \quad f(-1)=0 \quad (-1,0)}$

$\boxed{\text{MIN At } x=2 \quad f(2)=0 \quad (2,0)}$

Question 8. *(10 marks)*
Find the following information about $f(x) = 3x^4 + 4x^3$
-The intervals where it is increasing and where it is decreasing
-Any relative extrema
-The intervals where it is concave up and where it is concave down
-Any inflection points
-For 3 BONUS MARKS, find the x and y intercepts and sketch $f(x)$

$f'(x) = 12x^3 + 12x^2 = 12x^2(x+1)$

critical #s $x = 0$ $x = -1$

Interval	$(-\infty,-1)$	$(-1,0)$	$(0,\infty)$
test pt	-2	$-1/2$	1
sign of f'	$-$	$+$	$+$
f incr/decr	$\searrow$	$\nearrow$	$\nearrow$

MIN At $x=-1$ $f(-1) = 3-4 = -1$

$\boxed{\text{MIN } (-1,-1)}$

$f''(x) = 36x^2 + 24x = 12x(3x+2)$

$f''(x) = 0$ At $x = 0$ $x = -2/3$

Interval	$(-\infty,-2/3)$	$(-2/3,0)$	$(0,\infty)$
Test pt.	-2	$-1/2$	1
Sign of f''	$+$	$-$	$+$
concavity	$\cup$	$\cap$	$\cup$

inflection points At $x = -2/3$ $f(-2/3) = -16/27$
& $x = 0$ $f(0) = 0$

Infl pts $(0,0)$
$(-2/3, -16/27)$

Sketch: y-intercept $(0,0)$

x-intercept $0 = 3x^4 + 4x^3$

$= x^3(3x+4)$

$x=0$ & $x=-4/3$

$(0,0)$ $(-4/3, 0)$