TEST 3
DAWSON COLLEGE
201-171 - Mathematical Models

LAST NAME: SOLUTIONS

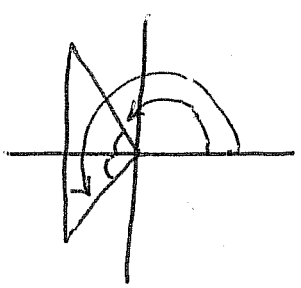
FIRST NAME:_____

STUDENT NUMBER:_____

INSTRUCTIONS
1- SHOW ALL YOUR WORK FOR FULL MARKS.
2- The test is marked out of 50 points.

Question 1. *(4 marks)*
Solve for x.
$\cos x = -0.23$

REF ANGLE $\cos^{-1}(0.23) = 76.7°$

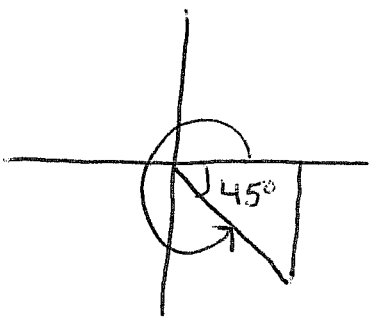


$x = 180 - 76.7° = \boxed{103.3°}$

$x = 180 + 76.7° = \boxed{256.7°}$

Question 2. *(4 marks)*
Solve for x.
$\tan x = -1$ and $\sin x < 0$

ref angle $\tan^{-1}(1) = 45°$



$x = 360° - 45°$
$= \boxed{315°}$

Question 3. *(4 marks)*
Solve for x.
$\sin^2 x - \sin x = 0$

$\sin x(\sin x - 1) = 0$

$\sin x = 0$ $\quad$ $\sin x = 1$

$\boxed{x = 0°, 90°, 180°}$

Question 4. *(6 marks)*
Solve for x.
$\cos^3 x - \frac{9}{16}\cos x = 0$.

$\cos x \left(\cos^2 x - \frac{9}{16}\right) = 0$

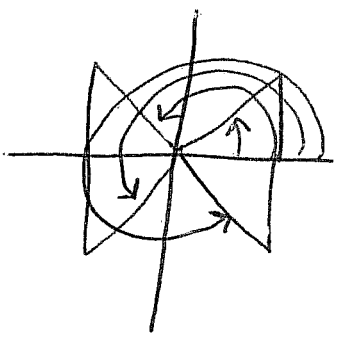
$\cos x = 0$

$\cos^2 x = \frac{9}{16}$

$\cos x = \pm \frac{3}{4}$

$x = 90°, 270°$

ref angle $\cos^{-1}(3/4) = 41.4°$



Solutions $x = 41.4°, 138.6°, 221.4°, 318.6°$

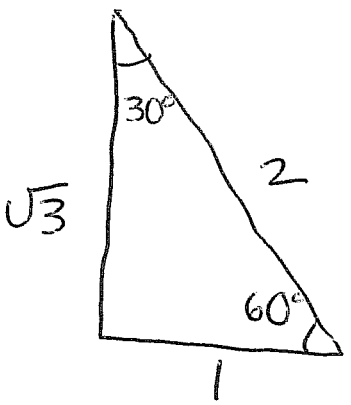
Question 5. *(6 marks)*

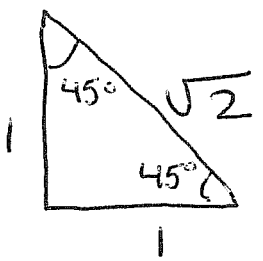
Draw the two "special triangles". Then give the **EXACT VALUES** of the following.

$\csc 30° =$ 2 $\sin 60° =$ $\sqrt{3}/2$

$\sin 45° =$ $1/\sqrt{2}$ $\cos 45° =$ $1/\sqrt{2}$

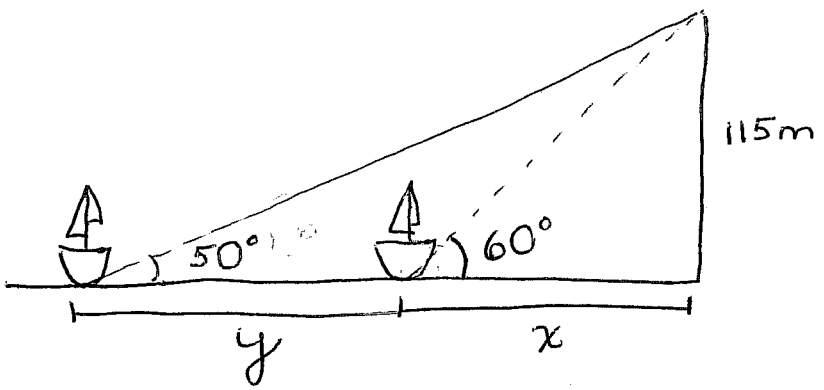
$\cot 30° =$ $\sqrt{3}$ $\tan 60° =$ $\sqrt{3}$





Question 5. *(6 marks)*

The angle of elevation from a boat to the top of a 115m cliff is 60°. A second boat is further away from the cliff and has an angle of elevation to the top of the cliff of 50°. What is the distance between the two boats?



$$\tan 60° = \frac{115}{x}$$

$$x = \frac{115}{\tan 60°} = 66.4\text{m}$$

$$x+y \Rightarrow \quad \tan 50° = \frac{115}{x+y}$$

$$x+y = \frac{115}{\tan 50°}$$

$$y = \frac{115}{\tan 50°} - x$$

$$= 96.5 - 66.4$$

$$= \boxed{30.1\text{m}}$$

Question 6. *(6 marks)*
Perform the given operations.
(a) $j^3\sqrt{-36}+3\sqrt{-4}+j^5$

(b) $\frac{3}{j}-\frac{2}{-1+j}$

(a) $-j(6j)+3(2j)+j$

$= -6j^2+7j$

$= \boxed{6+7j}$

(b) $\frac{3}{j}-\frac{2}{-1+j}$

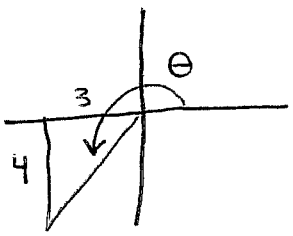
$= \frac{3}{j}\frac{(-j)}{(-j)}-\frac{2}{(-1+j)}\frac{(-1-j)}{(-1-j)}$

$= -3j-\frac{(-2-2j)}{2}$

$= -3j+1+j$

$= \boxed{1-2j}$

Question 7. *(4 marks)*
Express $-3-4j$ in **polar** and **exponential** form.



ref angle $\tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$

$\theta = 180^\circ + 53.1^\circ$

$= 233.1^\circ$

$r^2 = 3^2 + 4^2 \qquad r^2 = 25$

$r = 5$

polar $\quad 5(\cos 233.1^\circ + j\sin 233.1^\circ)$

exponential $\quad 5e^{233.1j}$

Question 8. *(4 marks)*
Express $4e^{75j}$ in rectangular form.

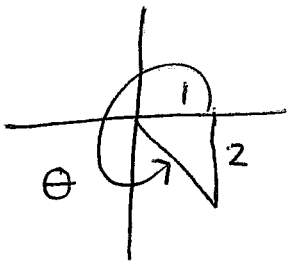
$4e^{75j} = 4(\cos 75^\circ + j\sin 75^\circ)$

$= 1.04 + j(3.86)$

$= \boxed{1.04 + 3.86j}$

Question 9. *(6 marks)*

Express $1-2j$ in exponential form, then express $\sqrt{1-2j}$ in rectangular form.



$\tan^{-1}(2) = 63.4°$

$\theta = 360° - 63.4°$

$= 296.6°$

$r^2 = 1^2 + 2^2 = 5$

$r = \sqrt{5}$

$1-2j = \sqrt{5}e^{296.6j}$

$\sqrt{1-2j} = (\sqrt{5})^{1/2}\left(e^{296.6j}\right)^{1/2}$

$= 1.5\, e^{148.3j}$

$= 1.5\,(\cos 148.3° + j\sin 148.3°)$

$= -1.28 + 0.79j$