Solutions - Absolute MAX/MIN BONUS

MATH 103

#15 $f(x) = x^2 - 2x - 3$ on $[-2,3]$

$f'(x) = 2x - 2$

critical # $x = 1$

$f(1) = -4$
$f(-2) = 5$
$f(3) = 0$

ABSOLUTE MIN	$(1,-4)$
ABSOLUTE MAX	$(-2,5)$

#17 $f(x) = -x^2 + 4x + 6$ on $[0,5]$

$f'(x) = -2x + 4$

critical # $x = 2$

$f(2) = -4 + 8 + 6 = 10$
$f(0) = 6$
$f(5) = 1$

Absolute MIN	$(5,1)$
Absolute MAX	$(2,10)$

#19 $f(x) = x^3 + 3x^2 - 1$ on $[-3,2]$

$f'(x) = 3x^2 + 6x$
$= 3x(x+2)$

critical #s $x = 0$ $x = -2$

$f(0) = -1$
$f(-2) = 3$
$f(-3) = -1$
$f(2) = 19$

Absolute Min	$(0,-1)$ & $(-3,-1)$
Absolute MAX	$(2,19)$

#21 $f(x) = 3x^4 + 4x^3$ ON $[-2, 1]$

$f'(x) = 12x^3 + 12x^2$
$= 12x^2(x+1)$ critical #s $x=0,\ x=-1$

$f(0) = 0$
$f(-1) = -1$
$f(-2) = 16$
$f(1) = 7$

Absolute minimum	$(-1,-1)$
Absolute MAXIMUM	$(-2,16)$

#23 $f(x) = \dfrac{x+1}{x-1}$ ON $[2,4]$

$f'(x) = \dfrac{(x-1)-(x+1)}{(x-1)^2} = \dfrac{-2}{(x-1)^2}$ critical # $x=+1$ Not in interval

$f(2) = \frac{3}{1} = 3$
$f(4) = 5/3$

Absolute minimum	$(4, 5/3)$
Absolute MAXIMUM	$(2,3)$

#25 $f(x) = 4x + \dfrac{1}{x}$ on $[1,3]$

$f'(x) = 4 - \dfrac{1}{x^2}$

$= \dfrac{4x^2-1}{x^2}$

$= \dfrac{(2x+1)(2x-1)}{x^2}$ critical #s

$x=0,\ x=-\frac{1}{2},\ x=\frac{1}{2}$

$x=0$ Not in interval
$x=-\frac{1}{2}$ Not in interval
$x=\frac{1}{2}$ " "

$f(1) = 4+1 = 5$
$f(3) = 37/3$

Absolute minimum	$(1,5)$
Absolute MAXIMUM	$(3, 37/3)$

27 $f(x) = \frac{1}{2}x^2 - 2\sqrt{x}$ ON $[0,3]$

$$f'(x) = x - 2\left(\tfrac{1}{2}x^{-1/2}\right)$$

$$= x - \frac{1}{\sqrt{x}} = \frac{x\sqrt{x} - 1}{\sqrt{x}} = \frac{x^{3/2} - 1}{\sqrt{x}}$$

critical # $x=1$
$x=0$

$f(1) = -3/2$
$f(0) = 0$
$f(3) = 1.04$

ABSOLUTE MINIMUM $(1, -3/2)$
ABSOLUTE MAXIMUM $(3, 1.04)$

31 $f(x) = x^{2/3} - 2x$ on $[0,3]$

$$f'(x) = \frac{2}{3}x^{-1/3} - 2$$

$\longrightarrow$ ALTERNATE METHOD for finding x

$$\frac{2}{3}x^{-1/3} = 2$$
$$x^{-1/3} = \frac{6}{2} = 3$$
$$x = (3)^{-3} = \frac{1}{27}$$

$$\frac{2}{3x^{1/3}} - 2 = 0$$

$$\frac{2 - 6x^{1/3}}{3x^{1/3}} = 0$$

critical #s
$x = 0$

$$2 - 6x^{1/3} = 0$$
$$6x^{1/3} = 2$$
$$x^{1/3} = 1/3$$
$$x = (1/3)^3 = 1/27$$

$f(1/27) = 1/27$
$f(0) = 0$
$f(3) = -3.92$

ABSOLUTE MINIMUM $(3, -3.92)$
ABSOLUTE MAXIMUM $(1/27, 1/27)$

33 $f(x) = x^{2/3}(x^2-4)$ on $[-1,3]$

$$f'(x) = \frac{2}{3}x^{-1/3}(x^2-4) + 2x(x^{2/3})$$

$$= \frac{2(x^2-4)}{3x^{1/3}} + 2x^{5/3}$$

$$= \frac{2(x^2-4) + 6x^{\frac{6}{3}}}{3x^{1/3}}$$

$$= \frac{2x^2-8+6x^2}{3x^{1/3}}$$

$$= \frac{8x^2-8}{3x^{1/3}} = \frac{8(x^2-1)}{3x^{1/3}}$$

critical # $x=0$
$x=\pm 1$

$f(0)=0$
$f(1)=-3$
$f(-1)=-3$
$f(3)=10.4$

$\boxed{\text{Absolute minimum } (1,-3) \;\&\; (-1,-3) \quad \text{Absolute maximum } (3,10.4)}$

35 $f(x) = \dfrac{x}{x^2+2}$ on $[-1,2]$

$$f'(x) = \frac{(x^2+2)-2x(x)}{(x^2+2)^2} = \frac{-x^2+2}{(x^2+2)^2}$$

critical #s
$x = \pm\sqrt{2}$

only $x=\sqrt{2}$ is in interval

$f(\sqrt{2}) = \sqrt{2}/4 = 0.35$
$f(-1) = -1/3$
$f(2) = 1/3$

$\boxed{\text{Absolute minimum } (-1,-1/3) \quad \text{Absolute maximum } (\sqrt{2},0.35)}$