

SOLUTIONS PRACTISE TEST #1

$$(1) \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 4} = \lim_{x \rightarrow 2} \frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}(x+2)} = \boxed{\frac{3}{4}}$$

$$(2) \lim_{x \rightarrow 1} \frac{x^2 + 2}{x - 1} = \frac{3}{0} \text{ does not exist}$$

$$(3) \lim_{x \rightarrow \infty} \frac{x^3 - 2x + 1}{3x^3 + 4x - 1} = \lim_{x \rightarrow \infty} \frac{x^3/x^3 - 2x/x^3 + 1/x^3}{3x^3/x^3 + 4x/x^3 - 1/x^3}$$

$$= \lim_{x \rightarrow \infty} \frac{1 - 2/x^2 + 1/x^3}{3 + 4/x^2 - 1/x^3}$$

$$= \boxed{1/3}$$

$$(4) \lim_{x \rightarrow 0} \frac{\frac{1}{x-1} + \frac{1}{x+1}}{x} = \lim_{x \rightarrow 0} \frac{\frac{x+1+x-1}{(x-1)(x+1)}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{2x}{(x-1)(x+1)x} = \frac{2}{-1(1)} = \boxed{-2}$$

$$(5) \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} x^2 - 1 = 0$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} x^2 - 2x + 1 = 0$$

so the limit exists $\lim_{x \rightarrow 1} f(x) = 0$

BUT $f(1) = 2$

since $\lim_{x \rightarrow 1} f(x) \neq f(1)$ not continuous

$$\begin{aligned}
 (6) \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - (x+h) + 1 - (x^2 - x + 1)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x - h + 1 - x^2 + x - 1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2xh + h^2 - h}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(2x + h - 1)}{h} = 2x - 1
 \end{aligned}$$

$$(7) \quad (a) \quad f(x) = x^2 - 3x + 5/2$$

$$f'(x) = 2x - 3$$

$$(b) \quad f(x) = \frac{1}{x^8} + x^{2/3}$$

$$= x^{-8} + x^{2/3}$$

$$f'(x) = -8x^{-9} + \frac{2}{3}x^{-1/3}$$

$$(c) \quad f(x) = \frac{2x^3 - x^{1/2}}{x^{3/2}}$$

$$= 2x^{3/2} - x^{-1}$$

$$f'(x) = 3x^{1/2} + x^{-2}$$

$$(d) \quad f(x) = x^{-7} + 281 \quad f'(x) = -7x^{-8}$$

$$(e) \quad f(x) = (x+1)(2x+3) = 2x^2 + 5x + 3$$

$$f'(x) = 4x + 5$$

$$(f) \quad f(x) = 3x^{-1/3} + x^{-5/2}$$

$$f'(x) = -x^{-4/3} - \frac{5}{2}x^{-7/2}$$