LAST NAME: SOLUTIONS

FIRST NAME: ______________

STUDENT NUMBER: ______________

TEST 1 (A)

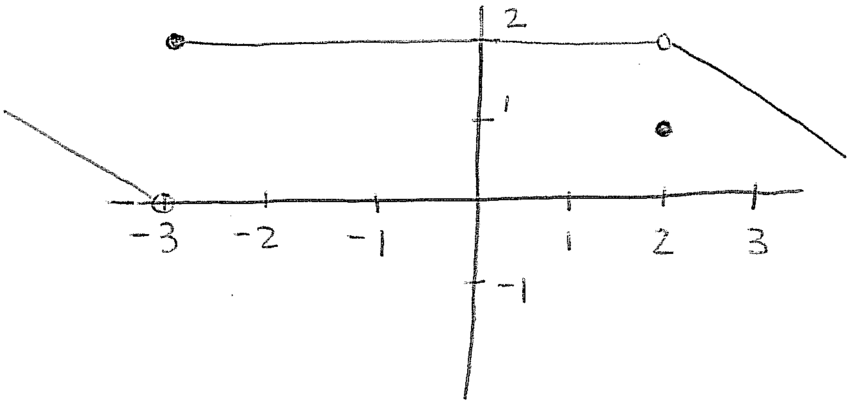
DAWSON COLLEGE

103-DW Section 4 - Calculus 1

Instructor: E. Richer

Date: Sept. 26th 2008

Question 1. *(2 marks each)*



Refer to the graph of the function $f(x)$ to determine whether each statement is true or false. Explain your answer.

(a) $\lim_{x\to -3^+} f(x) = 2$ TRUE

(b) $\lim_{x\to -3^-} f(x) = f(-3)$ FALSE $f(-3) = 2$ $\lim_{x\to -3^-} f(x) = 0$

(c) $\lim_{x\to 2^-} f(x) = 2$ TRUE

(d) $f(x)$ is continuous at $x = 2$ FALSE $\rightarrow \lim_{x\to 2} f(x) \neq f(2)$

(e) $\lim_{x\to 2} f(x)$ does not exist. FALSE

$\lim_{x\to 2} f(x) = 2$

Question 2.

Evaluate the following limits.

(a) *(2 marks)* $\lim_{x \to 2} \frac{x^2-5x+4}{x^2-2x-8}$

(b) *(3 marks)* $\lim_{x \to 0} \frac{\sqrt{x+5}-\sqrt{5}}{x}$

(c) *(3 marks)* $\lim_{x \to 0} \frac{\frac{1}{x+4}-\frac{1}{4}}{x}$

(d) *(2 marks)* $\lim_{x \to \infty} \frac{6x^4-5x^2+x-5}{2x^4+3x^2-7}$

(a) $\lim_{x \to 2} \frac{(x-4)(x-1)}{(x-4)(x+2)} = \lim_{x \to 2} \frac{x-1}{x+2} = \boxed{\frac{1}{4}}$

(b) $\lim_{x \to 0} \frac{\sqrt{x+5}-\sqrt{5}}{x} \left(\frac{\sqrt{x+5}+\sqrt{5}}{\sqrt{x+5}+\sqrt{5}} \right)$

$= \lim_{x \to 0} \frac{x+5-5}{x(\sqrt{x+5}+\sqrt{5})} = \lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+5}+\sqrt{5})}$

$= \lim_{x \to 0} \frac{1}{\sqrt{x+5}+\sqrt{5}} = \boxed{\frac{1}{\sqrt{5}+\sqrt{5}}}$

(c) $\lim_{x \to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$

$= \lim_{x \to 0} \frac{\frac{4-(x+4)}{4(x+4)}}{x} = \lim_{x \to 0} \frac{-\cancel{x}}{4(x+4)} \cdot \frac{1}{\cancel{x}}$

$= \lim_{x \to 0} \frac{-1}{4(x+4)} = \boxed{\frac{-1}{16}}$

(d) $\lim_{x \to \infty} \frac{6x^4/x^4 - 5x^2/x^4 + x/x^4 - 5/x^4}{2x^4/x^4 + 3x^2/x^4 - 7/x^4} = \lim_{x \to \infty} \frac{6 - \cancel{5/x^2}^{\to 0} + \cancel{1/x^3}^{\to 0} - \cancel{5/x^4}^{\to 0}}{2 + \cancel{3/x^2}^{\to 0} - \cancel{7/x^4}^{\to 0}}$

$= 6/\ = \boxed{3}$

Question 3. *(5 marks)*
Determine if the function $g(x)$ is continuous at $x = 2$. **Justify** your answer using the definition of continuity at a point.

$$g(x) = \begin{cases} 5 & x < 2 \\ 3x - 2 & x = 2 \\ 5 + 2x - x^2 & x > 2 \end{cases}$$

$$g(2) = 3(2) - 2$$
$$= 4$$

$$\lim_{x \to 2^-} g(x) = 5$$

$$\lim_{x \to 2^+} g(x) = \lim_{x \to 2} 5 + 2x - x^2$$
$$= 5 + 4 - 4 = 5$$

So lim exists

$$\lim_{x \to 2} g(x) = 5$$

$g(x)$ is not continuous at $x=2$
Because

$$g(2) \neq \lim_{x \to 2} g(x)$$
$$4 \neq 5$$

Question 4. *(10 marks)*
(a) Use the **limit definition** to find the derivative of $f(x) = -x^2 + 2x + 1$.
(b) Find the equation of the tangent line to $f(x)$ at the point $(2, 1)$.

(a) $$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{-(x+h)^2 + 2(x+h) + 1 - (-x^2+2x+1)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{-x^2} - 2xh - h^2 + \cancel{2x} + 2h + \cancel{1} + \cancel{x^2} - \cancel{2x} - \cancel{1}}{h}$$

$$= \lim_{h\to 0} \frac{-2xh - h^2 + 2h}{h} = \lim_{h\to 0} \frac{\cancel{h}(-2x - h + 2)}{\cancel{h}}$$

$$= \boxed{-2x + 2}$$

(b) slope at $(2,1)$

$x = 2$ $\quad f'(2) = -2(2) + 2$
$= -2$

$y = -2x + b$

$1 = -2(2) + b$

$5 = b$

$$\boxed{y = -2x + 5}$$

Question 5. (*2 marks* each)
Find the derivative of the following functions.
(a) $f(x) = -2x^6 + \frac{1}{x} - 2$
(b) $f(t) = 6t^{\frac{1}{3}} - 12t^{\frac{1}{2}}$
(c) $f(x) = (2x^{-5} + 3x^{\frac{1}{5}})(2x+1)$
(d) $f(x) = \frac{2}{x^4} - \frac{3x}{\sqrt{x}}$
(e) $f(t) = \frac{2t^3-5t^2+7t}{\sqrt[4]{t}}$

(a) $f(x) = -2x^6 + x^{-1} - 2$
$\boxed{f'(x) = -12x^5 - x^{-2}}$

(b) $\boxed{f'(t) = 2t^{-2/3} - 6t^{-1/2}}$

(c) $f(x) = 4x^{-4} + 2x^{-5} + 6x^{6/5} + 3x^{1/5}$
$\boxed{f'(x) = -16x^{-5} - 10x^{-6} + \frac{36}{5}x^{1/5} + 3/5\, x^{-4/5}}$

(d) $f(x) = 2x^{-4} - 3x^{1/2}$
$\boxed{f'(x) = -8x^{-5} - \frac{3}{2}x^{-1/2}}$

(e) $f(t) = 2t^{11/4} - 5t^{7/4} + 7t^{3/4}$
$\boxed{f'(t) = \frac{11}{2}t^{7/4} - \frac{35}{4}t^{3/4} + \frac{21}{4}t^{-1/4}}$

Question 6. (*5 marks*)
The stopping distance (in m) of a car t seconds after applying the breaks is given by the formula: $s(t) = -0.4t^3 - 0.6t^2 + 7.2t$.
(a) After applying the breaks, how long does it take before the car stops?
(b) How many meters does the car travel before stopping?

(a) THE CAR IS STOPPED when its velocity is 0, velocity = $s'(t)$

$$v(t) = -1.2t^2 - 1.2t + 7.2$$
$$= -1.2(t^2 + t - 6)$$
$$= -1.2(t+3)(t-2)$$
$$t = 2 \text{ or } \cancel{t = -3}$$

NO NEGATIVE

After $\boxed{2 \text{ seconds}}$

(b)
$$s(2) = -0.4(2)^3 - 0.6(2)^2 + 7.2(2)$$
$$= -3.2 - 2.4 + 14.4$$
$$= \boxed{8.8\text{m}}$$