Algebra 201-007-50 03
Test 2
October 24, 2008

Name: SOLUTIONS
Student Number:

1. Simplify, writing your answer with positive exponents only:

a) *(3 marks.)*

$$\frac{x^4 y^{-2} z^2 x}{y^{-6} x^2 z^7} = \frac{x^4 y^6 z^2 x}{y^2 x^2 z^7}$$

$$= \frac{x^{4+1} y^6 z^2}{y^2 x^2 z^7} = \frac{x^5 y^6 z^2}{y^2 x^2 z^7} = x^{5-2} y^{6-2} z^{2-7}$$

$$= x^3 y^4 z^{-5} = \frac{x^3 y^4}{z^5}$$

b) *(3 marks.)*

$$(-3a^{-7} b^3 c^{-2})^2 =$$

$$= (-3)^2 (a^{-7})^2 (b^3)^2 (c^{-2})^2$$

$$= 9 a^{-14} b^6 c^{-4}$$

$$= \frac{9 b^6}{a^{14} c^4}$$

c) *(3 marks.)*

$$\left(\frac{5x^5y^{-3}}{a^4y^3}\right)^{-2} = \left(\frac{a^4y^3}{5x^5y^{-3}}\right)^2$$

$$= \frac{(a^4y^3)^2}{(5x^5y^{-3})^2} = \frac{(a^4)^2(y^3)^2}{(5)^2(x^5)^2(y^{-3})^2} = \frac{a^8y^6}{25x^{10}y^{-6}}$$

$$= \frac{a^8y^6y^6}{25x^{10}} = \frac{a^8y^{12}}{25x^{10}}$$

d) *(2 marks.)*

$$\left(\frac{567x^8y^{98}z^{12}}{513x^{31}b^4z^{-344}}\right)^0 = \underline{1}$$

e) *(4 marks.)*

$$\frac{a^5b^{-4}(ab)^{-3}}{(a^{-2}b^2)^2a^3} = \frac{a^5}{b^4(ab)^3(a^{-2}b^2)^2a^3} =$$

$$= \frac{a^5}{b^4a^3b^3(a^{-2})^2(b^2)^2a^3} = \frac{a^5}{b^4a^3b^3a^{-4}b^4a^3} =$$

$$= \frac{a^5a^4}{a^3a^3b^4b^3b^4} = \frac{a^{5+4}}{a^{3+3}b^{4+3+4}} = \frac{a^9}{a^6b^{11}} = \frac{a^{9-6}}{b^{11}} = \frac{a^3}{b^{11}}$$

2. *(3 marks).* Subtract and simplify:

$$(13x^6 - 12x^3 + x^2 - 4x - 2) - (2x^6 - 5x^5 + x^3 + x^2 - 6)$$

$= 13x^6 - 12x^3 + x^2 - 4x - 2 - 2x^6 + 5x^5 - x^3 - x^2 + 6$

$= 11x^6 + 5x^5 - 13x^3 - 4x + 4$

3. Multiply and simplify:

a) *(3 marks.)*

$$(4x - 7)^2$$

$= (4x-7)(4x-7)$

$= 4x(4x-7) - 7(4x-7)$

$= 16x^2 - 28x - 28x + 49$

$= 16x^2 - 56x + 49$

b) *(4 marks.)*

$$(3x-8)(12x^2-6x+13)$$

$$= 3x(12x^2-6x+13) - 8(12x^2-6x+13)$$
$$= 36x^3 - 18x^2 + 39x - 96x^2 + 48x - 104$$
$$= 36x^3 - 114x^2 + 87x - 104$$

4. *(4 marks.)* Simplify:

$$6x(2x+1) - (2x+3)(2x-2)$$
$$= (12x^2+6x) - [2x(2x-2) + 3(2x-2)]$$
$$= 12x^2 + 6x - (4x^2 - 4x + 6x - 6)$$
$$= 12x^2 + 6x - (4x^2 + 2x - 6)$$
$$= 12x^2 + 6x - 4x^2 - 2x + 6$$
$$= 8x^2 + 4x + 6$$

5. *(5 marks).* Divide by long division and write what the following equals:

$$\frac{6x^2 - 19x + 30}{2x - 5}$$

$$2x-5 \;\overline{)\; 6x^2-19x+30} \qquad 3x - 2$$

$3x(2x-5) \longrightarrow -(6x^2 - 15x)$ ↓

$-4x + 30$

$-2(2x-5) \longrightarrow -(-4x + 10)$

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$$\therefore \frac{6x^2-19x+30}{2x-5} = 3x+2 + \frac{20}{2x-5}$$

6. *(6 marks).* Divide by long division and indicate the quotient and the remainder:

$$\frac{x^4 + 4x^2 - 3x + 9}{x^2 + 5x - 1}$$

x^2+5x-1) $x^4+0x^3+4x^2-3x+9$ $\quad x^2-5x+30$

$x^2(x^2+5x-1) \longrightarrow -(x^4+5x^3-x^2)$

$-5x^3+5x^2-3x$

$-5x(x^2+5x-1) \longrightarrow -(-5x^3-25x^2+5x)$

$30x^2-8x+9$

$30(x^2+5x-1) \longrightarrow -(30x^2+150x-30)$

$-158x+39$

QUOTIENT: $x^2-5x+30$

REMAINDER: $-142x+39$

7. *(4 marks.)* Factor by grouping:

$$4x^3 + 12x^2 + 3x + 9$$

$= 4x^2(x+3) + 3(x+3)$

$= (4x^2+3)(x+3)$

8. Factor completely using the appropriate method:

a) *(2 marks.)* (check your answer for marks).

$$44x^5 - 11x^2$$

$$= 11x^2(4x^3 - 1)$$

CHECK: $11x^2(4x^3-1) = 44x^5 - 11x^2$

b) *(2 marks.)*

$$x^5y^5 - x^3y^3 + x^2y^4$$

$$= x^2y^3(x^3y^2 - x + y)$$

c) *(4 marks.)* (check your answer for marks).

$$x^2 + 10x + 16$$

$$= (x+8)(x+2)$$

$A \cdot B = 16$

$A + B = 10$

$A = 8, B = 2$

CHECK: $(x+8)(x+2) = x(x+2) + 8(x+2)$

$= x^2 + 2x + 8x + 16$

$= x^2 + 10x + 16$

d) *(3 marks.)*

$$x^2 + 20x + 91$$

$$= (x+7)(x+13)$$

$A \cdot B = 91$

$A + B = 20$

$A = 7, \quad B = 13$

e) *(5 marks.)* (check your answer for marks).

$$8x^2 - 19x + 6 =$$

$(8)(6) = 48$

$A \cdot B = 48$

$A + B = -19$

$A = -3 \quad B = -16$

$$= 8x^2 - 3x - 16x + 6 = x(8x-3) - 2(8x-3)$$

$$= (8x-3)(x-2)$$

CHECK: $(8x-3)(x-2) = 8x(x-2) - 3(x-2)$

$$= 8x^2 - 16x - 3x + 6 = 8x^2 - 19x + 6$$

f) *(3 marks.)*

$$8x^3 - 125 =$$

$$= (2x)^3 - (5)^3$$

$$= (2x-5)((2x)^2 + (2x)(5) + (5)^2)$$

$$= (2x-5)(4x^2 + 10x + 25)$$

g) *(5 marks.)*

$$x^4 - 16$$

$$= (x^2)^2 - (4)^2$$

$$= (x^2+4)(x^2-4)$$

$$= (x^2+4)\left[(x)^2-(2)^2\right]$$

$$= (x^2+4)(x+2)(x-2)$$

h) *(4 marks.)*

$$10x^4 - 230x^3 + 420x^2$$

$$= 10x^2(x^2 - 23x + 42)$$

$$= 10x^2(x-21)(x-2)$$

$A \cdot B = 42$

$A + B = -23$

$A = -21 \quad B = -2$