

Algebra 201-007-50 03

Test 3

November 21, 2008

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Student ID: _____

1. Solve for x

a) (3 marks).

$$x^2 - 2x = 48$$

$$x^2 - 2x - 48 = 0$$

$$(x-8)(x+6) = 0$$

$$\begin{array}{l} \downarrow \\ x-8=0 \\ x=8 \end{array} \quad \begin{array}{l} \searrow \\ x+6=0 \\ x=-6 \end{array}$$

b) (4 marks).

$$2x^3 = \del{2x^3} 4x^2 + 16x$$

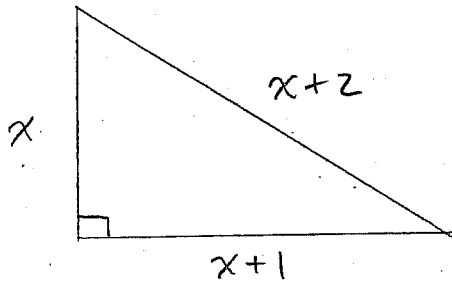
$$2x^3 - 4x^2 - 16x = 0$$

$$2x(x^2 - 2x^2 - 8) = 0$$

$$2x(x-4)(x+2) = 0$$

$$\begin{array}{l} \downarrow \\ 2x=0 \\ x=0 \end{array} \quad \begin{array}{l} \searrow \\ x-4=0 \\ x=4 \end{array} \quad \begin{array}{l} \searrow \\ x+2=0 \\ x=-2 \end{array}$$

2. (4 marks). Solve for x :



$$x^2 + (x+1)^2 = (x+2)^2$$

$$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$$

$$2x^2 + 2x + 1 - x^2 - 4x - 4 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$\downarrow$$

$$x-3=0$$

$$x=3$$

$$\downarrow$$

$$x+1=0$$

$$x=-1$$

CAN'T HAVE NEGATIVE LENGTH.

$$\boxed{x=3}$$

3. (5 marks). Divide and simplify:

$$\frac{x^2 + 2x - 15}{x^2 - 49} \div \frac{x^2 + 7x - 30}{x^2 + 4x - 21}$$

$$= \frac{x^2 + 2x - 15}{x^2 - 49} \cdot \frac{x^2 + 4x - 21}{x^2 + 7x - 30}$$

$$= \frac{(x+5)(x-3)}{(x+7)(x-7)} \cdot \frac{(x+7)(x-3)}{(x+10)(x-3)}$$

$$= \frac{(x+5)(x-3)}{(x-7)(x+10)}$$

4. (5 marks). Subtract and simplify:

$$\frac{3}{x^2 - 13x + 40} - \frac{4}{x^2 - 12x + 32}$$

$$= \frac{3}{(x-8)(x-5)} - \frac{4}{(x-8)(x-4)} = \frac{3}{(x-8)(x-5)} \cdot \frac{x-4}{x-4} - \frac{4}{(x-8)(x-4)} \cdot \frac{x-5}{x-5}$$

$$= \frac{3(x-4)}{(x-8)(x-5)(x-4)} - \frac{4(x-5)}{(x-8)(x-4)(x-5)} = \frac{(3x-12) - (4x-20)}{(x-8)(x-4)(x-5)}$$

$$= \frac{-x+8}{(x-8)(x-4)(x-5)} = \frac{-1}{(x-4)(x-5)}$$

5. (6 marks). Simplify the complex fraction:

$$\frac{1 + \frac{2}{x}}{1 - \frac{4}{x^2}}$$

$$= \frac{1 \cdot \frac{x}{x} + \frac{2}{x}}{1 \cdot \frac{x^2}{x^2} - \frac{4}{x^2}} = \frac{\frac{x}{x} + \frac{2}{x}}{\frac{x^2}{x^2} - \frac{4}{x^2}} = \frac{\frac{x+2}{x}}{\frac{x^2-4}{x^2}}$$

$$= \frac{x+2}{x} \div \frac{x^2-4}{x^2} = \frac{x+2}{x} \cdot \frac{x^2}{x^2-4}$$

$$= \frac{x+2}{x} \cdot \frac{x^2}{(x+2)(x-2)} = \frac{x}{x-2}$$

6. Solve for x :

a) (5 marks).

$$1 + \frac{3}{x-4} = \frac{33-9x}{x^2-9x+20}$$

$$1 + \frac{3}{x-4} = \frac{33-9x}{(x-5)(x-4)}$$

$$(x-5)(x-4)(1) + (x-5)(x-4) \frac{3}{x-4} = (x-5)(x-4) \frac{33-9x}{(x-5)(x-4)}$$

$$x^2 - 9x + 20 + 3x - 15 = 33 - 9x$$

$$x^2 - 6x + 5 = 33 - 9x$$

$$x^2 + 3x - 28 = 0$$

$$(x+7)(x-4) = 0$$

$$(x+7)(x-4) = 0$$

$$x+7=0$$

$$x=-7$$

VALID

$$x-4=0$$

$$x=4$$

EXTRANEUS

b) (4 marks).

$$\frac{3}{5+x} + \frac{1}{5-x} = \frac{4}{x^2-25}$$

$$\frac{3}{5+x} - \frac{1}{x-5} = \frac{4}{(x+5)(x-5)}$$

$$(x+5)(x-5) \frac{3}{x+5} - (x+5)(x-5) \frac{1}{x-5} = (x+5)(x-5) \frac{4}{(x+5)(x-5)}$$

$$3x-15 - (x+5) = 4$$

$$2x-20 = 4$$

$$2x = 24$$

$$x = 12$$

VALID

$$\therefore \boxed{x=12}$$

$$\boxed{x=-7}$$

7. (5 marks). The sum of a number and its reciprocal is $\frac{50}{7}$. Find the number.

LET x BE THE NUMBER.

$$x + \frac{1}{x} = \frac{50}{7}$$

$$7x \cdot x + 7x \cdot \frac{1}{x} = 7x \cdot \frac{50}{7}$$

$$7x^2 + 7 = 50x$$

$$7x^2 - 50x + 7 = 0$$

$$7x^2 - 49x - x + 7 = 0$$

$$7x(x-7) - 1(x-7)$$

$$(7x-1)(x-7) = 0$$

$$\begin{array}{l} \swarrow \\ 7x-1=0 \\ 7x=1 \\ x=\frac{1}{7} \\ \text{VALID} \end{array} \quad \begin{array}{l} \searrow \\ x-7=0 \\ x=7 \\ \text{VALID} \end{array}$$

$$x = \frac{1}{7}$$

VALID

$$\boxed{x = \frac{1}{7}, 7}$$

8. Solve for x in the proportion:

a) (2 marks).

$$\frac{2}{9} = \frac{3}{x}$$

$$2x = 3 \cdot 9$$

$$x = \frac{27}{2}$$

b) (4 marks).

$$\frac{2x+7}{4} = \frac{1}{x}$$

$$(2x+7)x = 1 \cdot 4$$

$$2x^2 + 7x = 4$$

$$2x^2 + 7x - 4 = 0$$

$$2x^2 + 8x - x - 4 = 0$$

$$2x(x+4) - 1(x+4) = 0$$

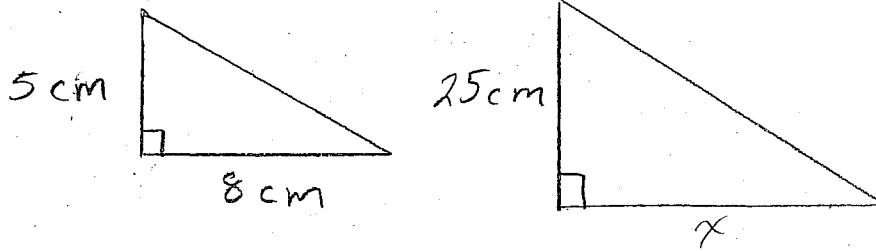
$$(2x-1)(x+4) = 0$$

$$\begin{array}{l} \swarrow \\ 2x-1=0 \\ 2x=1 \\ x=\frac{1}{2} \end{array} \quad \begin{array}{l} \searrow \\ x+4=0 \\ x=-4 \end{array}$$

$$x = \frac{1}{2}$$

$$\therefore \boxed{x = \frac{1}{2}, -4}$$

9. (5 marks). Given that the following are similar triangles, find the area of the triangle on the right:



$$\frac{x}{8} = \frac{25}{5}$$

$$\therefore \text{Area} = \frac{40 \cdot 25}{2} = \frac{1000}{2} = 500 \text{ cm}^2$$

$$5x = 25 \cdot 8$$

$$x = \frac{200}{5} = 40 \text{ cm}$$

10. Simplify:

a) (2 marks).

$$\begin{aligned} \sqrt{80} &= \sqrt{16 \cdot 5} = \sqrt{16} \cdot \sqrt{5} \\ &= 4\sqrt{5} \end{aligned}$$

b) (2 marks).

$$\begin{aligned} \sqrt[3]{\frac{8}{27}} &= \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3} \end{aligned}$$

c) (2 marks).

$$\begin{aligned} &5\sqrt{2} \cdot 4\sqrt{32} \\ &= 5 \cdot 4 \sqrt{2} \sqrt{32} \\ &= 20 \sqrt{2 \cdot 32} = 20 \sqrt{64} \\ &= 20 \cdot 8 = 160 \end{aligned}$$