

Algebra 201-007-50 C1

Test 2

October 23, 2008

Name: SOLUTIONS

Student Number:

1. Simplify, writing your answer with positive exponents only:

a) (3 marks.)

$$\begin{aligned} & (-2x^{-4}y^6z^{-2})^4 \\ &= (-2)^4 (x^{-4})^4 (y^6)^4 (z^{-2})^4 \\ &= 16 x^{-16} y^{24} z^{-8} = \frac{16y^{24}}{x^{16}z^8} \end{aligned}$$

b) (3 marks.)

$$\begin{aligned} & \frac{a^5 b^{-3} c^2 a}{b^{-2} a^3 b^7} \\ &= \frac{a^5 b^2 c^2 a}{b^3 a^3 b^7} = \frac{a^{5+1} b^2 c^2}{a^3 b^{7+3}} = \frac{a^6 b^2 c^2}{a^3 b^{10}} \\ &= a^{6-3} b^{2-10} c^2 = a^3 b^{-8} c^2 = \frac{a^3 c^2}{b^8} \end{aligned}$$

c) (4 marks.)

$$\frac{(xy)^{-3} x^4 y^{-6}}{(x^2 y^{-2})^2 x^3} = \frac{x^4}{(xy)^3 (x^2 y^{-2})^2 y^6 x^3}$$

$$= \frac{x^4}{x^3 y^3 (x^2)^2 (y^{-2})^2 y^6 x^3} = \frac{x^4}{x^3 x^3 y^3 x^4 y^{-4} y^6} = \frac{x^4 y^4}{x^3 x^3 x^4 y^3 y^2}$$

$$= \frac{x^4 y^4}{x^{3+3+4} y^{3+2}} = \frac{x^4 y^4}{x^{10} y^5} = x^{4-10} y^{4-5} = x^{-6} y^{-5} = \frac{1}{x^6 y^5}$$

d) (3 marks.)

$$\left(\frac{4a^{-2}b^3}{a^4d^{-3}}\right)^{-2} = \left(\frac{a^4d^{-3}}{4a^{-2}b^3}\right)^2$$

$$= \frac{(a^4d^{-3})^2}{(4a^{-2}b^3)^2} = \frac{(a^4)^2 (d^{-3})^2}{(4)^2 (a^{-2})^2 (b^3)^2} = \frac{a^8 d^{-6}}{16 a^{-4} b^6}$$

$$= \frac{a^8 a^4}{16 b^6 d^6} = \frac{a^{8+4}}{16 b^6 d^6} = \frac{a^{12}}{16 b^6 d^6}$$

e) (2 marks.)

$$\left(\frac{14x^4y^{38}z^{124}}{5413x^{11}b^{24}z^{-34}}\right)^0 = 1$$

2. (3 marks). Subtract and simplify:

$$\begin{aligned} & (17x^6 - 14x^4 + 3x^2 + x - 2) - (2x^6 + 5x^4 + x^3 + x - 6) \\ &= 17x^6 - 14x^4 + 3x^2 + x - 2 - 2x^6 - 5x^4 - x^3 - x + 6 \\ &= 15x^6 - 19x^4 - x^3 + 3x^2 + 4 \end{aligned}$$

3. Multiply and simplify:

a) (4 marks.)

$$\begin{aligned} & (2x - 7)(24x^2 - 13x + 12) \\ &= 2x(24x^2 - 13x + 12) - 7(24x^2 - 13x + 12) \\ &= 48x^3 - 26x^2 + 24x - 168x^2 + 91x - 84 \\ &= 48x^3 - 194x^2 + 115x - 84 \end{aligned}$$

b) (3 marks.)

$$\begin{aligned}(3x-5)^2 &= (3x-5)(3x-5) \\ &= 3x(3x-5) - 5(3x-5) \\ &= 9x^2 - 15x - 15x + 25 \\ &= 9x^2 - 30x + 25\end{aligned}$$

4. (4 marks.) Simplify:

$$\begin{aligned}6x(2x+1) - (2x+3)(2x-2) &= (12x^2 + 6x) - [2x(2x-2) + 3(2x-2)] \\ &= (12x^2 + 6x) - (4x^2 - 4x + 6x - 6) \\ &= 12x^2 + 6x - (4x^2 + 2x - 6) \\ &= 12x^2 + 6x - 4x^2 - 2x + 6 \\ &= 8x^2 + 4x + 6\end{aligned}$$

5. (5 marks). Divide by long division and write what the following equals:

$$\frac{12x^2 - 11x + 15}{3x - 5}$$

$$\begin{array}{r} 4x + 3 \\ 3x - 5 \overline{) 12x^2 - 11x + 15} \\ \underline{4x(3x - 5) \rightarrow -(12x^2 - 20x)} \\ -9x + 15 \\ \underline{(3x - 5) \rightarrow -(9x - 15)} \\ 30 \end{array}$$

$$\therefore \frac{12x^2 - 11x + 15}{3x - 5} = 4x + 3 + \frac{30}{3x - 5}$$

6. (6 marks). Divide by long division and indicate the quotient and the remainder:

$$\frac{x^4 + 5x^2 - 2x + 4}{x^2 + 4x - 1}$$

$$\begin{array}{r}
 x^2 - 4x + 22 \\
 \hline
 x^2 + 4x - 1 \overline{) x^4 + 0x^3 + 5x^2 - 2x + 4} \\
 \underline{x^2(x^2 + 4x - 1) \rightarrow -(x^4 + 4x^3 - x^2)} \quad \downarrow \\
 -4x^3 + 6x^2 - 2x \quad \downarrow \\
 \underline{-4x(x^2 + 4x - 1) \rightarrow -(-4x^3 - 16x^2 + 4x)} \quad \downarrow \\
 22x^2 - 6x + 4 \\
 \underline{22(x^2 + 4x - 1) \rightarrow -(22x^2 + 88x - 22)} \\
 -94x + 26
 \end{array}$$

QUOTIENT: $x^2 - 4x + 22$

REMAINDER: $-94x + 26$

7. Factor completely using the appropriate method:

a) (2 marks.) (check your answer for marks).

$$44x^5 - 11x^2$$

$$11x^2(4x^3 - 1)$$

CHECK: $11x^2(4x^3 - 1) = 44x^5 - 11x^2$

b) (2 marks.)

$$x^5y^5 - x^3y^3 + x^2y^4 =$$

$$= x^2y^3(x^3y^2 - x + y)$$

c) (4 marks.) (check your answer for marks).

$$x^2 - 2x - 15 \\ = (x - 5)(x + 3)$$

$$A \cdot B = -15$$

$$A + B = -2$$

$$A = -5 \quad B = 3$$

CHECK: $(x - 5)(x + 3) = x(x + 3) - 5(x + 3)$
 $= x^2 + 3x - 5x - 15$
 $= x^2 - 2x - 15$

d) (5 marks.) (check your answer for marks).

$$2x^2 - 13x + 18 =$$

$$\begin{matrix} \swarrow & \nearrow \\ (2) & (18) \end{matrix} = 36$$

$$= 2x^2 - 9x - 4x + 18$$

$$= x(2x - 9) - 2(2x - 9) = (2x - 9)(x - 2)$$

$$A \cdot B = 18$$

$$A + B = -13$$

$$A = -9 \quad B = -4$$

CHECK: $(2x - 9)(x - 2) = 2x(x - 2) - 9(x - 2) = 2x^2 - 4x - 9x + 18$
 $= 2x^2 - 13x + 18$

e) (4 marks.)

$$\begin{aligned} & x^3 - 125y^3 \\ &= (x)^3 - (5y)^3 \\ &= (x - 5y)(x^2 + (x)(5y) + (5y)^2) \\ &= (x - 5y)(x^2 + 5xy + 25y^2) \end{aligned}$$

f) (5 marks.)

$$\begin{aligned} & x^4 - 16 \\ &= (x^2)^2 - (4)^2 \\ &= (x^2 + 4)(x^2 - 4) \\ &= (x^2 + 4)[(x)^2 - (2)^2] \\ &= (x^2 + 4)[(x + 2)(x - 2)] \\ &= (x^2 + 4)(x + 2)(x - 2) \end{aligned}$$

g) (4 marks.)

$$10x^8 + 20x^7 - 990x^6$$

$$10x^6 (x^2 + 2x - 99)$$

$$10x^6 (x+11)(x-9)$$

$$A \cdot B = -99$$

$$A+B=2$$

$$A=11, B=-9$$

8. Solve each equation:

a) (5 marks.) (check your answer for marks).

$$x^2 - 7x = 30$$

$$x^2 - 7x - 30 = 0$$

$$(x+3)(x-10) = 0$$

$$x+3=0$$

$$x=-3$$

$$x-10=0$$

$$x=10$$

$$x = -3, 10$$

$$A \cdot B = -30$$

$$A+B = -7$$

$$A=3, B=-10$$

CHECK: $x = -3$

$$(-3)^2 - 7(-3) = 30?$$

$$9 + 21 = 30?$$

$$30 = 30 \text{ TRUE}$$

$x = 10$

$$(10)^2 - 7(10) = 30?$$

$$100 - 70 = 30?$$

$$30 = 30 \text{ TRUE}$$

b) (5 marks.)

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$$27x^2 - 3x - 2 = 0$$

$$27x^2 + 6x - 9x - 2 = 0$$

$$3x(9x+2) - 1(9x+2) = 0$$

$$(3x-1)(9x+2) = 0$$

$$\swarrow$$

$$3x-1=0$$

$$3x=1$$

$$x=\frac{1}{3}$$

$$\searrow$$

$$9x+2=0$$

$$9x=-2$$

$$x=-\frac{2}{9}$$

$$A \cdot B = -34$$

$$A + B = -3$$

$$A = 6 \quad B = -9$$

$$x = -\frac{2}{9}, \frac{1}{3}$$

9. (6 marks.) The sum of the squares of three consecutive integers is 110. Find the integers.

LET x BE THE FIRST INTEGER $\left\{ \begin{array}{l} 5 \\ 6 \text{ OR} \\ 7 \end{array} \right.$ $\left\{ \begin{array}{l} -7 \\ -6 \\ -5 \end{array} \right.$

THEN $x+1$ IS THE SECOND INTEGER

AND $x+2$ IS THE THIRD INTEGER

$$(x)^2 + (x+1)^2 + (x+2)^2 = 110$$

$$x^2 + (x^2 + 2x + 1) + (x^2 + 4x + 4) = 110$$

$$3x^2 + 6x + 5 = 110$$

$$3x^2 + 6x - 105 = 0$$

$$3(x^2 + 2x - 35) = 0$$

$$3(x+7)(x-5) = 0$$

$$\swarrow$$

$$x+7=0$$

$$x=-7$$

$$\searrow$$

$$x-5=0$$

$$x=5$$

$$A \cdot B = -35$$

$$A + B = 2$$

$$A = 7 \quad B = -5$$

AND SO THE THREE INTEGERS ARE 5, 6 AND 7 OR -7, -6, -5.