

Algebra 201-007-50 C1

Test 3

November 20, 2008

Name: SOLUTIONS
Student ID: _____

1. (5 marks). Divide and simplify:

$$\begin{aligned} & \frac{x^2 - 3x - 10}{x^2 + 2x} \div \frac{x^2 + 8x}{x^2 + 3x - 40} \\ = & \frac{x^2 - 3x - 10}{x^2 + 2x} \cdot \frac{x^2 + 3x - 40}{x^2 + 8x} \\ = & \frac{(x-5)(x+2)}{x(x+2)} \cdot \frac{(x+8)(x-5)}{x(x+8)} \\ = & \frac{(x-5)(x-5)}{(x)(x)} = \frac{(x-5)^2}{x^2} \end{aligned}$$

2. (5 marks). Subtract and simplify:

$$\frac{4}{x^2 - 6x - 27} - \frac{2}{x^2 - 12x + 27}$$

$LCD = (x-9)(x+3)(x-3)$

$$= \frac{4}{(x-9)(x+3)} - \frac{2}{(x-9)(x-3)}$$

$$= \frac{4}{(x-9)(x+3)} \cdot \frac{x-3}{x-3} - \frac{2}{(x-9)(x-3)} \cdot \frac{x+3}{x+3}$$

$$= \frac{4x-12}{(x-9)(x+3)(x-3)} - \frac{2x+6}{(x-9)(x-3)(x+3)} = \frac{(4x-12) - (2x+6)}{(x-9)(x-3)(x+3)}$$

$$= \frac{2x-18}{(x-9)(x-3)(x+3)} = \frac{2(x-9)}{(x-9)(x-3)(x+3)} = \frac{2}{(x-3)(x+3)}$$

3. (6 marks). Simplify the complex fraction:

$$\frac{1 - \frac{4}{x^2}}{1 - \frac{2}{x}}$$

$$= \frac{1 - \frac{4}{x^2}}{1 - \frac{2}{x}} = \frac{\frac{x^2}{x^2} - \frac{4}{x^2}}{\frac{x}{x} - \frac{2}{x}} = \frac{\frac{x^2-4}{x^2}}{\frac{x-2}{x}} = \frac{x^2-4}{x^2} \cdot \frac{x}{x-2}$$

$$= \frac{x^2-4}{x^2} \cdot \frac{x}{x-2} = \frac{(x+2)(x-2)}{x^2} \cdot \frac{x}{x-2} = \frac{x+2}{x}$$

4. Solve for x :

a) (5 marks).

$$1 - \frac{5}{x-3} = \frac{38-11x}{x^2-7x+12}$$

$$1 - \frac{5}{x-3} = \frac{38-11x}{(x-4)(x-3)}$$

$$\angle \text{CD} = (x-3)(x-4)$$

$$(x-4)(x-3) \cdot 1 - (x-4)(x-3) \cdot \frac{5}{x-3} = (x-4)(x-3) \frac{38-11x}{(x-4)(x-3)}$$

$$x^2 - 7x + 12 - 5(x-4) = 38 - 11x \rightarrow (x-3)(x+2) = 0$$

$$x^2 - 12x + 32 = 38 - 11x$$

$$x^2 - x - 6 = 0$$

$$\begin{aligned} x-3 &= 0 \\ x &= 3 \\ \text{EXTRANEOUS} \end{aligned}$$

$$\begin{aligned} x+2 &= 0 \\ x &= -2 \\ \text{VALID} \end{aligned}$$

$$\boxed{x = -2}$$

b) (4 marks).

$$\frac{2}{x+3} + \frac{1}{3-x} = \frac{1}{x^2-9}$$

$$\frac{2}{x+3} - \frac{1}{x-3} = \frac{1}{(x+3)(x-3)}$$

$$\angle \text{CD} = (x+3)(x-3)$$

$$(x+3)(x-3) \cdot \frac{2}{x+3} - (x+3)(x-3) \cdot \frac{1}{x-3} = (x+3)(x-3) \cdot \frac{1}{(x+3)(x-3)}$$

$$2(x-3) - (x+3) = 1$$

$$2x - 6 - x - 3 = 1$$

$$x - 9 = 1$$

$$x = 10$$

VALID

$$\boxed{x = 10}$$

5. (5 marks). The difference of the reciprocal of a number and $\frac{5}{9}$ is $-\frac{41}{90}$. Find the number.

LET x BE THE NUMBER

$$LCD = 90x$$

$$\frac{1}{x} - \frac{5}{9} = -\frac{41}{90}$$

$$90x \cdot \frac{1}{x} - 90x \cdot \frac{5}{9} = -90x \cdot \frac{41}{90}$$

$$90 - 50x = -41x$$

$$90 = 9x$$

$$\boxed{10 = x} \text{ VALID}$$

6. Solve for x in the proportion:

a) (2 marks).

$$\frac{5}{3} = \frac{x}{7}$$

$$7 \cdot 5 = 3x$$

$$\frac{35}{3} = x$$

b) (4 marks).

$$\frac{x}{2} = \frac{1}{3x+5}$$

$$x(3x+5) = 2$$

$$3x^2 + 5x = 2$$

$$3x^2 + 5x - 2 = 0$$

$$3x^2 + 6x - x - 2 = 0$$

$$3x(x+2) - 1(x+2) = 0$$

$$(x+2)(3x-1) = 0$$

$$x+2=0$$

$$x = -2$$

$$3x-1=0$$

$$3x = 1$$

$$x = \frac{1}{3}$$

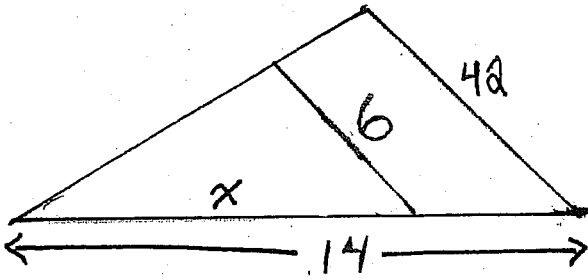
$$\boxed{x = -2, \frac{1}{3}}$$

$$A \cdot B = -6$$

$$A + B = 5$$

$$A = 6 \quad B = -1$$

7. (5 marks). Solve for x in the following similar triangles:



$$\frac{x}{14} = \frac{6}{42}$$

$$42x = 6 \cdot 14$$

$$x = \frac{84}{42}$$

$$\therefore x = 2$$

8. Simplify:

a) (2 marks).

$$\begin{aligned} & \sqrt{48} \\ &= \sqrt{16 \cdot 3} \\ &= \sqrt{16} \sqrt{3} \\ &= 4\sqrt{3} \end{aligned}$$

b) (3 marks).

$$\begin{aligned} & \sqrt[3]{\frac{64}{27}} \\ &= \frac{\sqrt[3]{64}}{\sqrt[3]{27}} = \frac{4}{3} \end{aligned}$$

c) (4 marks).

$$\begin{aligned} & (\sqrt{6} + \sqrt{3})^2 \\ &= (\sqrt{6} + \sqrt{3})(\sqrt{6} + \sqrt{3}) = \sqrt{6} \cdot \sqrt{6} + \sqrt{6} \sqrt{3} + \sqrt{3} \sqrt{6} + \sqrt{3} \sqrt{3} \\ &= 6 + \sqrt{6 \cdot 3} + \sqrt{3 \cdot 6} + 3 \\ &= 9 + \sqrt{18} + \sqrt{18} \\ &= 9 + 2\sqrt{18} = 9 + 2\sqrt{9 \cdot 2} \\ &= 9 + 2\sqrt{9} \sqrt{2} = 9 + 2 \cdot 3\sqrt{2} = 9 + 6\sqrt{2} \end{aligned}$$

9. Rationalize and simplify (in lowest terms):

a) (3 marks).

$$\frac{7}{\sqrt{7}} = \frac{7}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{7\sqrt{7}}{7} = \sqrt{7}$$

b) (5 marks).

$$\begin{aligned} & \frac{5\sqrt{5}}{\sqrt{15} + \sqrt{5}} \\ = & \frac{5\sqrt{5}}{\sqrt{15} + \sqrt{5}} \cdot \frac{\sqrt{15} - \sqrt{5}}{\sqrt{15} - \sqrt{5}} = \frac{5\sqrt{5} \cdot \sqrt{15} - 5\sqrt{5} \cdot \sqrt{5}}{15 - 5} \\ = & \frac{5\sqrt{75} - 5 \cdot 5}{10} = \frac{5\sqrt{25 \cdot 3} - 25}{10} = \frac{5 \cdot 5\sqrt{3} - 25}{10} \\ = & \frac{25\sqrt{3} - 25}{10} = \frac{25(\sqrt{3} - 1)}{10} = \frac{5(\sqrt{3} - 1)}{2} \end{aligned}$$

10. Simplify:

a) (3 marks).

$$\begin{aligned} & 27^{\frac{2}{3}} \\ = & \left(\sqrt[3]{27}\right)^2 = (3)^2 = 9 \end{aligned}$$

b) (3 marks).

$$\begin{aligned} & 2^{\frac{7}{4}} \cdot 2^{\frac{1}{4}} \\ = & 2^{\frac{7}{4} + \frac{1}{4}} = 2^{\frac{8}{4}} = 2^2 = 4 \end{aligned}$$