Algebra 201-007-50 C2
Test 2
October 22, 2008

Name: SOLUTIONS

Student Number:

1. (5 marks). Solve the following systems of equations:

$$\text{a) } 3x - 11y = 43 \quad \text{①}$$
$$2x - 7y = 27 \quad \text{②}$$

① $\times 2$: $6x - 22y = 86$

② $\times 3$ $\{ 6x - 21y = 81 \;)$

_____

$-y = 5$

$\therefore \boxed{y = -5}$

② $2x - 7(-5) = 27$

$2x + 35 = 27$

$2x = 27 - 35$

$2x = -8$

$\boxed{x = -4}$

3. *(6 marks).* During a matinee show a movie theater had 32 adults and 52 children and took in $432 in admission. How much do adult tickets and children's tickets cost if adult tickets are $3 more than children's tickets?

LET x BE THE PRICE OF ADULT TICKETS

LET y BE THE PRICE OF CHILDREN'S TICKETS

$\therefore$ $32x + 52y = 432$ ①

② $7 = y + 3$

$x = y + 3 \Rightarrow x - y = 3$ ②

$7 - 3 = y$

$4 = y$

② × 52 : $\begin{array}{r} 32x + 52y = 432 \\ + \quad 52x - 52y = 156 \\ \hline 84x \qquad\quad = 588 \end{array}$

$x = 7$

$\therefore$ ADULT TICKETS ARE $7 AND CHILDRENS TICKETS ARE $4

4. Simplify, writing your answer with positive exponents only:

a) *(2 marks.)*

$$\frac{x^5 \, y^{-6} \, y^3 \, b}{x^{-2} \, a^4 \, b^7}$$

$$= \frac{x^5 x^2 y^3 b}{y^6 a^4 b^7} = \frac{x^{5+2} y^{3-6} b^{1-7}}{a^4} = \frac{x^7 y^{-3} b^{-6}}{a^4}$$

$$= \frac{x^7}{a^4 b^6 y^3}$$

b) *(2 marks.)*

$$(-2\,x^4 b^{-6} c^{-7})^2$$

$$= (-2)^2 (x^4)^2 (b^{-6})^2 (c^{-7})^2$$

$$= 4\,x^8 b^{-12} c^{-14}$$

$$= \frac{4x^8}{b^{12} c^{14}}$$

c) *(3 marks.)*

$$\left(\frac{a^4 b^7}{3\,c^5 d^{-8}}\right)^{-2} =$$

$$= \left(\frac{3c^5 d^{-8}}{a^4 b^7}\right)^2 = \frac{(3c^5 d^{-8})^2}{(a^4 b^7)^2} = \frac{(3)^2 (c^5)^2 (d^{-8})^2}{(a^4)^2 (b^7)^2}$$

$$= \frac{9\,c^{10} d^{-16}}{a^8 b^{14}} = \frac{9\,c^{10}}{a^8 b^{14} d^{16}}$$

d) *(3 marks.)*

$$\frac{(x\,y)^{-3} x^4 y^{-6}}{(x^2 y^{-2})^2 x^3}$$

$$= \frac{x^4}{(xy)^3 (x^2 y^{-2})^2 x^3 y^6} = \frac{x^4}{x^3 y^3 (x^2)^2 (y^{-2})^2 x^3 y^6}$$

$$= \frac{x^4}{x^3 y^3 (x^2)^2 (y^{-2})^2 x^3 y^6} = \frac{x^4}{x^3 y^3 x^4 y^{-4} x^3 y^6} = \frac{x^4 y^4}{x^3 x^4 x^3 y^3 y^6}$$

$$= \frac{x^4 y^4}{x^{(3+4+3)} y^{(3+6)}} = \frac{x^4 y^4}{x^{10} y^9} \qquad 4 = x^{4-10} y^{4-9} = x^{-6} y^{-5} = \frac{1}{x^6 y^5}$$

e) *(2 marks.)*

$$\left(\frac{144534\,x^5\,y^{19}\,z^45}{5432\,a^{11}\,b^{70}\,c^24}\right)^0 = 1$$

5. *(3 marks).* Subtract and simplify:

$$(15x^5 - 10x^3 - 2x^2 + x - 5) - (3x^5 + 2x^4 - x^2 + x + 1)$$

$= 15x^5 - 10x^3 - 2x^2 + x - 5 - 3x^5 - 2x^4 + x^2 - x - 1$

$= 12x^5 - 2x^4 - 10x^3 - x^2 - 6$

6. Multiply and simplify:

a) *(3 marks.)*

$$(5x - 7)^2$$

$= (5x-7)(5x-7) = \quad 5 \times (5x-7) - 7(5x-7)$

$= 25x^2 - 35x - 35x + 49 = 25x^2 - 70x + 49$

b) *(5 marks.)*

$$(7x - 4)(49x^2 + 28x + 16)$$

$= 7x(49x^2 + 28x + 16) - 4(49x^2 + 28x + 16)$

$= 343x^3 + 196x^2 + 112x - 196x^2 - 112x - 64$

$= 343x^3 - 64$

6

c) *(5 marks.)*

$$6x(x + 1) - (2x + 2)(2x - 2)$$

$$= 6x^2 + 6x - [2x(2x - 2) + 2(2x - 2)]$$

$$= 6x^2 + 6x - (4x^2 - 4x + 4x - 4)$$

$$= 6x^2 + 6x - (4x^2 - 4)$$

$$= 6x^2 + 6x - 4x^2 + 4$$

$$= 2x^2 + 6x + 4$$

7. *(5 marks).* Divide by long division and write what the following equals:

$$\frac{14x^2 - 3x - 39}{7x + 2}$$

$$
\begin{array}{r}
2x - 1 \\
7x + 2 \overline{)14x^2 - 3x - 39}
\end{array}
$$

$2(7x+2) \rightarrow -(14x^2 + 4x)$

$-7x - 39$

$-1(7x + 2) \rightarrow -(-7x - 2)$

-37

$$\frac{14x^2 - 3x - 39}{7x + 2} = 2x - 1 - \frac{37}{7x + 2}$$

7

8. *(6 marks).* Divide by long division and indicate the quotient and the remainder:

$$\frac{x^4 + 6x^2 - 3x + 1}{x^2 + 3x - 1}$$

$$
\begin{array}{r}
x^2 - 3x + 16 \\
x^2 + 3x - 1\ \overline{\big)\ x^4 + 0x^3 + 6x^2 - 3x + 1}
\end{array}
$$

$x^2(x^2 + 3x - 1) \rightarrow -(x^4 + 3x^3 - x^2)$

$\qquad\qquad\qquad -3x^3 + 7x^2 - 3x$

$-3x(x^2 + 3x - 1) \longrightarrow -(-3x^3 - 9x^2 + 3x)$

$\qquad\qquad\qquad\qquad\qquad 16x^2 - 6x + 1$

$16(x^2 + 3x - 1) \longrightarrow -(16x^2 + 48x - 16)$

$\qquad\qquad\qquad\qquad\qquad\qquad -54x + 17$

QUOTIENT: $x^2 - 3x - 2$

REMAINDER: -1

9. Factor completely:

a) *(1 mark.)*

$$11x^4 - 33x^3$$

$$= 11x^3(x - 3)$$

8

b) *(2 marks.)*

$$x^2y^3 - x^5y^2 + x^2y$$

$$= x^2y(y^2 - x^3y + 1)$$

c) *(4 marks.)* (check your answer for marks).

$$x^2 + 11x - 12$$

$$= (x-1)(x+12)$$

$A \cdot B = -12$

$A + B = 11$

$A = -1 \quad B = 12$

CHECK: $(x-1)(x+12) = x(x+12) - 1(x+12)$

$$= x^2 + 12x - x - 12$$

$$= x^2 + 11x - 12 \quad \checkmark$$

d) *(4 marks.)*

$$10x^2 - 29x + 10 =$$

$(10)(10) = 100$

$A \cdot B = 100$

$A + B = -29$

$A = -4 \quad B = -25$

$$= 10x^2 - 4x - 25x + 10$$

$$= 2x(5x - 2) - 5(5x - 2)$$

$$= (5x-2)(2x-5)$$

e) *(4 marks.)* (check your answer for marks).

$$64x^2 - 9 =$$

$$= (8x)^2 - (3)^2 = (8x+3)(8x-3)$$

CHECK: $(8x+3)(8x-3) = 8x(8x-3) + 3(8x-3)$

$$= 64x^2 - 24x + 24x - 9$$

$$= 64x^2 - 9$$

f) *(4 marks.)*

$$4x^2 + 20x + 21 =$$

$$(4)(21) = 84$$

$A \cdot B = 84$
$A + B = 20$
$A = 14, \; B = 6$

$$= 4x^2 + 14x + 6x + 21$$

$$= 2x(2x+7) + 3(2x+7)$$

$$= (2x+7)(2x+3)$$

h) *(3 marks.)*

$$x^3 + 125 = (x)^3 + (5)^2$$

$$= (x+5)((x)^2 - (x)(5) + (5)^2)$$

$$= (x+5)(x^2 - 5x + 25)$$

i) *(5 marks.)*

$$16x^4 - 81$$

$$= (4x^2)^2 - (9)^2$$

$$= (4x^2 + 9)(4x^2 - 9)$$

$$= (4x^2 + 9)\left[(2x)^2 - (3)^2\right]$$

$$= (4x^2 + 9)(2x + 3)(2x - 3)$$