Algebra 201-007-50 C2
Test 3
November 19, 2008

Name: SOLUTIONS

Student ID:

1. Solve for x

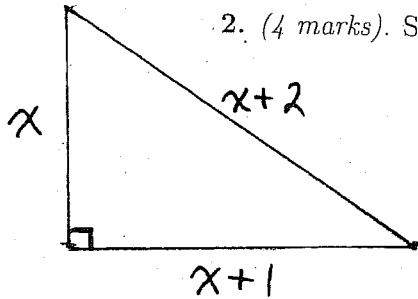a) *(3 marks).*

$$x^2 = 2x + 35$$

$x^2 - 2x - 35 = 0$

$(x-7)(x+5) = 0$

$x - 7 = 0 \qquad x + 5 = 0$

$x = 7 \qquad\qquad x = -5$

$\therefore x = -5, 7$

b) *(4 marks).*

$$3x^3 + 21x^2 = 132x$$

$3x^3 + 21x^2 - 132x = 0$

$3x(x^2 + 7x - 44) = 0$

$3x(x+11)(x-4) = 0$

$3x = 0 \qquad x + 11 = 0 \qquad x - 4 = 0$

$x = 0 \qquad\quad x = -11 \qquad\quad x = 4$

$\therefore x = -11, 0, 4$

2. *(4 marks).* Solve for x:

$$x^2 + (x+1)^2 = (x+2)^2$$

$$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$$

$$2x^2 + 2x + 1 = x^2 + 4x + 4$$

$$2x^2 + 2x + 1 - x^2 - 4x - 4 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x - 3 = 0 \qquad x + 1 = 0$$
$$x = 3 \qquad x = -1$$
$$\text{CAN'T HAVE NEGATIVE LENGTH}$$

$$\boxed{x = 3}$$

3. *(5 marks).* Divide and simplify:

$$\frac{x^2 + 3x - 70}{5x^2 + 6x - 8} \div \frac{x^2 - 7x}{x^2 + 9x + 14}$$

$5x^2 + 6x - 8$
$5x^2 + 10x - 4x - 8$
$5x(x+2) - 4(x+2)$
$= (x+2)(5x-4)$

$$= \frac{x^2 + 3x - 70}{5x^2 + 6x - 8} \cdot \frac{x^2 + 9x + 14}{x^2 - 7x}$$

$$= \frac{(x-7)(x+10)}{(x+2)(5x-4)} \cdot \frac{(x+2)(x+7)}{x(x-7)}$$

$$= \frac{(x+10)(x+7)}{x(5x-4)}$$

4. *(5 marks).* Subtract and simplify:

$$\frac{7}{x^2 - 5x - 6} - \frac{3}{x^2 - 9x + 18}$$

$$= \frac{7}{(x-6)(x+1)} - \frac{3}{(x-3)(x-6)} \qquad LCD = (x-6)(x+1)(x-3)$$

$$= \frac{7}{(x-6)(x+1)} \cdot \frac{x-3}{x-3} - \frac{3}{(x-3)(x-6)} \cdot \frac{x+1}{x+1}$$

$$= \frac{(7x-21)-(3x+3)}{(x-6)(x+1)(x-3)} = \frac{4x-24}{(x-6)(x+1)(x-3)} = \frac{4(x-6)}{(x-6)(x+1)(x-3)}$$

$$= \frac{4}{(x+1)(x-3)}$$

5. *(6 marks).* Simplify the complex fraction:

$$\frac{2 + \frac{8}{x}}{1 - \frac{16}{x^2}}$$

$$= \frac{2 \cdot \frac{x}{x} + \frac{8}{x}}{1 \cdot \frac{x^2}{x^2} - \frac{16}{x^2}} = \frac{\frac{2x+8}{x}}{\frac{x^2-16}{x^2}} = \frac{2x+8}{x} \div \frac{x^2-16}{x}$$

$$= \frac{2x+8}{x} \cdot \frac{x^2}{x^2-16} = \frac{2(x+4)}{x} \cdot \frac{x^2}{(x+4)(x-4)} = \frac{2x}{x-4}$$

6. Solve for x:

a) *(5 marks).*

$$1 + \frac{5}{x-4} = \frac{8x+13}{x^2+x-20}$$

$$1 + \frac{5}{x-4} = \frac{8x+15}{(x+5)(x-4)} \qquad LCD = (x+5)(x-4)$$

$$(x+5)(x-4)(1) + (x+5)(x-4)\cdot\frac{5}{x-4} = (x+5)(x-4)\frac{8x+13}{(x+5)(x-4)}$$

$$x^2+x-20 + 5(x+5) = 8x+13$$

$$x^2+x-20+5x+25 = 8x+13$$

$$x^2+6x+5 = 8x+13$$

$$x^2+6x+5-8x-13 = 0$$

$$x^2-2x-8=0$$
$$(x-4)(x+2)=0$$
$$x=4 \qquad x=-2$$
$$\text{EXTRANEOUS} \qquad \text{VALID}$$

$$\therefore \boxed{x=-2}$$

b) *(4 marks).*

$$\frac{2}{x+2} + \frac{1}{2-x} = \frac{10}{x^2-4}$$

$$\frac{2}{x+2} - \frac{1}{x-2} = \frac{10}{(x+2)(x-2)} \qquad LCD = (x+2)(x-2)$$

$$(x+2)(x-2)\cdot\frac{2}{x+2} - (x+2)(x-2)\frac{1}{x-2} = (x+2)(x-2)\frac{10}{(x+2)(x-2)}$$

$$2(x-2) - (x+2) = 10$$

$$2x-4 -x-2 =10$$

$$x-6 = 10$$

$$x=16$$
$$\text{VALID}$$

$$\therefore \boxed{x=16}$$

7. *(5 marks).* The sum of seven times the reciprocal of a number and $\frac{5}{8}$ is $\frac{3}{2}$. Find the number.

LET x BE THE NUMBER

$7. \quad \frac{1}{x} + \frac{5}{8} = \frac{3}{2}$

$\frac{7}{x} + \frac{5}{8} = \frac{3}{2} \quad LCD = 8x$

$8x \cdot \frac{7}{x} + 8x \cdot \frac{5}{8} = 8x \cdot \frac{3}{2}$

$56 + 5x = 12x$

$56 = 7x$

$x = 8$

$\therefore$ THE NUMBER IS 8.

8. Solve for x in the proportion:

a) *(2 marks).*

$$\frac{8}{x} = \frac{7}{5}$$

$8 \cdot 5 = 7 \cdot x$

$40 = 7x$

$\frac{40}{7} = x$

b) *(4 marks).*

$$\frac{5}{2x - 2} = \frac{3}{2x + 6}$$

$5(2x + 6) = 3(2x - 2)$
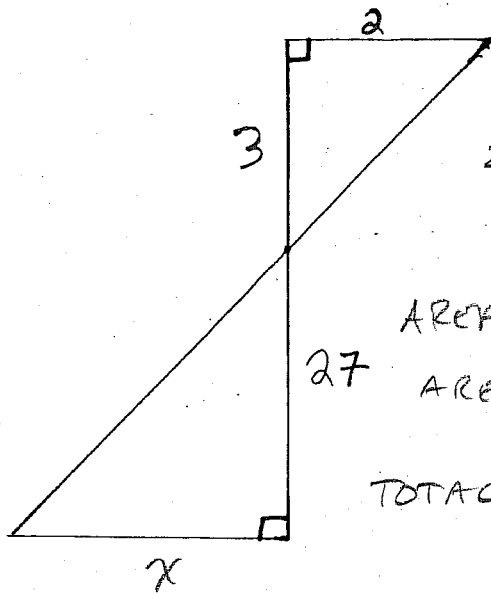
$10x + 30 = 6x - 6$

$10x - 6x = -6 - 30$

$4x = -36$

$x = -9$

9. *(5 marks).* Given that the following are similar triangles, find the area of the following figure:

$$\frac{2}{x} = \frac{3}{27}$$

$$2 \cdot 27 = 3x$$

$$18 = x$$

AREA OF SMALL TRIANGLE $= \frac{2 \cdot 3}{2} = 3$

AREA OF LARGE TRIANGLE $= \frac{18 \cdot 27}{2} = 243$

TOTAL AREA OF FIGURE $= 246$

10. Simplify:

a) *(2 marks).*

$$\sqrt{180}$$

$$= \sqrt{36 \cdot 5}$$

$$= \sqrt{36} \cdot \sqrt{5}$$

$$= 6\sqrt{5}$$

b) *(2 marks).*

$$2\sqrt{3} \cdot 5\sqrt{48}$$

$$= 2 \cdot 5 \sqrt{3} \sqrt{48}$$

$$= 10\sqrt{3 \cdot 48}$$

$$= 10\sqrt{144}$$

$$= 10 \cdot 12 = 120$$

c) *(2 marks).*

$$\sqrt[3]{\frac{27}{64}}$$

$$= \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$$