QUIZ #8

FIND THE POINTS OF INFLECTION AND DISCUSS THE CONCAVITY OF THE FUNCTION $f(x) = \frac{x}{x^2+1}$

$$f'(x) = \frac{(1)(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{x^2-1-2x^2}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2}$$

$$f''(x) = \frac{(-2x)(x^2+1)^2 - (-x^2+1)[2(x^2+1)(2x)]}{(x^2+1)^4}$$

$$= \frac{-2x(x^2+1)^2 + (4x^3+4x)(x^2+1)}{(x^2+1)^4}$$

$$= \frac{(x^2+1)[-2x(x^2+1) + (4x^3+4x)]}{(x^2+1)^4}$$

$$= \frac{-2x^3-2x+4x^3-4x}{(x^2+1)^3}$$

$$= \frac{2x^3-6x}{(x^2+1)^3} = \frac{2x(x^2-3)}{(x^2+1)^3}$$

$\therefore f''(x) = 0$ WHEN $x = 0, \pm\sqrt{3}$ ($f''(x)$ EXISTS FOR ALL REAL NUMBERS)

	$-\infty < x < -\sqrt{3}$	$-\sqrt{3} < x < 0$	$0 < x < \sqrt{3}$	$\sqrt{3} < x < \infty$
$f''(x)$	$-$	$+$		
CONCAVITY	∩	∪	∩	∪

$-\sqrt{3}$ $\quad$ 0 $\quad$ $\sqrt{3}$

CONCAVE DOWN ON $(-\infty, -\sqrt{3})$ AND $(0, \sqrt{3})$

CONCAVE UP ON $(-\sqrt{3}, 0)$ AND $(\sqrt{3}, \infty)$

INFLECTION POINTS: $(-\sqrt{3}, -\frac{\sqrt{3}}{4})$, $(0,0)$, $(\sqrt{3}, \frac{\sqrt{3}}{4})$

2) FIND THE LIMIT: $\lim_{x \to -\infty} \frac{x}{\sqrt{x^2 - x}}$

$$\lim_{x \to -\infty} \frac{x}{\sqrt{x^2 - x}} = \lim_{x \to -\infty} \frac{\frac{x}{x}}{\frac{\sqrt{x^2 - x}}{x}}$$

$$= \lim_{x \to -\infty} \frac{1}{\frac{\sqrt{x^2 - x}}{-\sqrt{x^2}}} = \lim_{x \to -\infty} \frac{1}{-\sqrt{\frac{x^2 - x}{x^2}}}$$

$$= \lim_{x \to +\infty} -\frac{1}{\sqrt{1 - \frac{1}{x}}} = \frac{-1}{\sqrt{1 - 0}} = -1$$