Calculus 201-NYA-05 C3
Quiz 9
November 15, 2008

Name: SOLUTIONS
Student ID:

1. *(10 marks).* Sketch the graph of f given that:

$$f(x) = \frac{x^2+1}{x^2-9} \quad f'(x) = \frac{-20x}{(x^2-9)^2} \quad f''(x) = \frac{60(x^2+3)}{(x^2-9)^3}$$

Clearly identify:
a)The x and y intercepts
b)Any horizontal and vertical asymptotes
c)The intervals where the function is increasing and decreasing
d)Any relative extrema
e)The Intervals where the graph of f is concave up and where it is concave down
f)Any points of inflection
g)Any symmetry that the graph has Label the graph with all relevant information.

LET $y = f(x) = \dfrac{x^2+1}{x^2-9}$

a) DOMAIN: ALL REAL NUMBERS EXCEPT $x = \pm 3$

NO x-int: $y \neq 0$

y-int: $x = 0$

$y = \dfrac{0^2+1}{0^2-9} = \dfrac{-1}{9}$

$\boxed{\therefore \text{ y-int is } (0, -\tfrac{1}{9})}$

b) $\lim\limits_{x\to\infty} \dfrac{x^2+1}{x^2-9} = \lim\limits_{x\to\infty} \dfrac{\frac{x^2+1}{x^2}}{\frac{x^2-9}{x^2}}$

$= \lim\limits_{x\to\infty} \dfrac{1+\frac{1}{x^2}}{1-\frac{9}{x^2}} = 1 \quad \therefore \boxed{\text{HA. } y=1}$

$\lim\limits_{x\to-\infty} \dfrac{x^2+1}{x^2-9} = \lim\limits_{x\to\infty} \dfrac{1+\frac{1}{x^2}}{1-\frac{9}{x^2}} = 1$

$\therefore \boxed{\text{HA. } y=1}$

POSSIBLE V.A'S $x = \pm 3$

$\lim\limits_{x\to3^-} \dfrac{x^2+1}{x^2-9} = \text{"}\dfrac{10}{0^-}\text{"} = -\infty$ } VA AT $x=3$

$\lim\limits_{x\to3^+} \dfrac{x^2+1}{x^2-9} = \text{"}\dfrac{10}{0^+}\text{"} = \infty$

$\lim\limits_{x\to-3^-} \dfrac{x^2+1}{x^2-9} = \text{"}\dfrac{10}{0^+}\text{"} = \infty$ } VA AT $x=-3$

$\lim\limits_{x\to-3^+} \dfrac{x^2+1}{x^2-9} = \text{"}\dfrac{10}{0^-}\text{"} = -\infty$

c) $f'(x) = 0$ WHEN $x = 0$

$f'(x)$ DOES NOT EXIST AT $x = \pm 3$

INTERVAL	$-\infty < x < -3$	$-3 < x < 0$	$0 < x < 3$	$3 < x < -\infty$
SIGN OF $f'(x)$	+	+	−	−
CONCLUSION	INCREASING	INCREASING	DECREASING	DECREASING

$-3 \quad 0 \quad 3$

∴ RELATIVE MAXIMUM $(0, 0)$

INCREASING ON $(-\infty, -3)$ AND $(-3, 0)$ DECREASING ON $(0, 3)$ AND $(3, 0)$

d) $f''(x) \neq 0$, $f''(x)$ DOES NOT EXIST WHEN $x = \pm 3$

INTERVAL	$-\infty < x < 3$	$-3 < x < 3$	$3 < x < \infty$
SIGN OF $f''(x)$	+	—	+
CONCLUSION	CONCAVE UP	CONCAVE DOWN	CONCAVE UP

$-3 \quad 3$

CONCAVE UP ON $(-\infty, -3)$ AND $(3, \infty)$

CONCAVE DOWN ON $(-3, 3)$

e) NO INFLECTION POINTS

f) $f(-x) = \dfrac{(-x)^2+1}{(-x)^2+9} = \dfrac{x^2+1}{x^2+9} = f(x)$ SYMMETRY! f IS EVEN.

GRAPH!