Total Cost, Total Revenue, Profit, Supply and Demand

Yann Lamontagne

Fall 2008

Total Cost, Total Revenue, and Profit:

- The *profit* is defined as

  $$P(x) \quad = \quad R(x) - C(x)$$

  where $R(x)$ is the *total revenue* from the sales of x units, $C(x)$ is the *total cost* of production for x units.

- *Revenue*, $R(x)$, is usually defined by the equation

  $$R(x) = \text{price per unit} \times \text{number of units}$$

- *Total Cost*, $C(x)$, is defined by the equation

  $$C(x) = \text{variable costs} + \text{fixed costs}$$

  where the *variable costs* depend on the number of units produced.

Total Cost, Total Revenue, and Profit: Example

- The ‘Are You Ready to Revolution’ print shop sells propaganda pamphlets. The pamphlets are sold for 2\$ each. The cost of production is 38\$ plus 0.10\$ per pamphlet. Write the total cost function, total revenue function, and profit function.

$$
\begin{aligned}
C(x) &= \text{variable costs} + \text{fixed costs} \\
&= 0.1x + 38 \\
R(x) &= \text{price per unit} \times \text{number of units} \\
&= 2x \\
P(x) &= R(x) - C(x) \\
&= 2x - (0.1x + 38) \\
&= 1.9x - 38
\end{aligned}
$$

Marginals

- The rate of change in profit with respect to the units produced and sold, is the slope of the profit function, and is called *marginal profit* ($\overline{\text{MP}}$). From the previous example: the profit function is $P(x) = 1.9x - 38$, hence $\overline{\text{MP}} = 1.9$.

- The rate of change in cost with respect to the units produced, is the slope of the cost function, and is called *marginal cost* ($\overline{\text{MC}}$). From the previous example: the cost function is $P(x) = 0.1x + 38$, hence $\overline{\text{MC}} = 0.1$.

- The rate of change in revenue with respect to the units produced and sold, is the slope of the revenue function, and is called *marginal revenue* ($\overline{\text{MR}}$). From the previous example: the cost function is $R(x) = 2x$, hence $\overline{\text{MR}} = 2$.

Break-Even Analysis

- The *break-even point* is the point at which the cost and revenue are equal. Hence when the profit function, $P(x) = 0$.

- We can use a graph to represent the break-even point. Graphing the cost function, $C(x)$, and the revenue function, $R(x)$, the intersection of the two functions is the break-even point. The x-component of the intersection represents the number of units needed to be sold before break even is attained. The y-component represents the revenue needed to break-even.

- **Example:** Finding the break-even point.

$$\begin{aligned} P(x) &= 0 \\ 1.9x - 38 &= 0 \\ x &= 20 \end{aligned}$$

Therefore the break-even point will be when 20 pamphlets are sold. All copies sold after the break-even point will generate profit since $R(x) > C(x)$.

Supply, Demand, and Market Equilibrium

- *Market equilibrium* is when the quantity of commodity demanded is equal to the quantity supplied

$$\text{Demand} = \text{Supply}$$

- The *law of demand* states that demand is related to price. The demand will rise as the price decreases and demand will decrease as the price increases.

- The *law of supply* states that supply is related to price. The supply will increase as the price increases and supply will decrease as the price decreases.

- The equations for demand and supply will have the price, p, and quantity, q variables.

- By convention the functions that represent demand and supply are functions with respect to supply.

- The solution to the system generated with the demand and supply function is the market equilibrium, which defines the *equilibrium price* and *equilibrium quantity*.

Supply, Demand, and Market Equilibrium: Example

- Find the market equilibrium for the following supply and demand functions:

$$\begin{aligned} \text{Demand:} \quad & 2p + 6q = 34 \\ \text{Supply:} \quad & p - 2q = 2 \end{aligned}$$

Solving by substitution:
Isolating p from the supply equation:

$$p = 2q + 2$$

Substituting p in the demand equation and solving for q:

$$\begin{aligned} 2p + 6q &= 34 \\ 2(2q+2) + 6q &= 34 \\ q &= 3 \end{aligned}$$

hence

$$\begin{aligned} p &= 2q + 2 \\ &= 2(3) + 2 \\ &= 8 \end{aligned}$$

Therefore $(q, p) = (3, 8)$.

Supply, Demand, and Market Equilibrium: Example

- The 'Agrarian Bike Shop' will buy 10 bicycles from a factory if the price is $70\$$ but will only buy 5 if the price is $120\$$. The factory is willing to sell 1 bike if the price is $50\$$ and 31 if the price is $200\$$. Assuming the resulting supply and demand functions are linear, find the market equilibrium point.

- Finding the demand function:

$$p = m_d q + b_d$$

where m_d is the slope and b_d the y-intercept.

$$\begin{aligned} m_d &= \frac{\Delta p}{\Delta q} \\ &= \frac{p_2 - p_1}{q_2 - q_1} \\ &= \frac{120 - 70}{5 - 10} = -10 \\ p &= -10q + b_d \\ 70 &= -10(10) + b_d \\ 170 &= b_d \end{aligned}$$

Therefore the demand equation is $p = -10q + 170$.

- Finding the supply function:

$$p = m_S q + b_S$$

where m_S is the slope and b_S the y-intercept.

$$\begin{aligned}
m_S &= \frac{\Delta p}{\Delta q} \\
&= \frac{p_2 - p_1}{q_2 - q_1} \\
&= \frac{200 - 50}{31 - 1} = 5 \\
p &= 5q + b_S \\
50 &= 5(1) + b_S \\
45 &= b_S
\end{aligned}$$

Therefore the supply equation is $p = 5q + 45$.

- Finding the equilibrium point:

$$\begin{aligned}
\text{Demand:} &\quad p = -10q + 170 \\
\text{Supply:} &\quad p = 5q + 45
\end{aligned}$$

Making the two equations equal.

$$
\begin{aligned}
-10q+170 &= 5q+45 \\
125 &= 15q \\
\frac{25}{3} &= q
\end{aligned}
$$

$$
\begin{aligned}
p &= 5\left(\frac{25}{3}\right)+45 \\
p &= 86.67\$
\end{aligned}
$$

Therefore the equilibrium is at $(q,p) = \left(\frac{25}{3}, 86.67\$\right)$. Hence the Agrarian Bike Shop is willing to buy 8.3 bikes for $86.67\$$ and the factory is willing to supply that amount for that price.