

Test 1

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (2 marks) 3 is 20% of what number?

$$\begin{aligned} \text{Percentage} &= \text{Base (rate)} \\ 3 &= \text{Base (0.2)} \\ 15 &= \text{Base} \end{aligned}$$

Question 2. (1 mark each) Simplify and write all answer without using exponents and decimals:

a.

$$-(-2)^3 = -(-\cancel{2})^3 = -(-8) = 8$$

b.

$$\left(\frac{-2}{3}\right)^4 = \frac{(-2)^4}{3^4} = \frac{16}{81}$$

c.

$$\left(\frac{5}{-2}\right)^{-3} = \left(\frac{-2}{5}\right)^3 = \frac{(-2)^3}{5^3} = \frac{-8}{125}$$

d.

$$(257)^0 = 1$$

Question 3. (2 marks each) Simplify and write all answers so that only positive exponents remain:

a.

$$\left(\frac{y^{-3}}{y^{-4}}\right)^2 = \frac{(y^{-3})^2}{(y^{-4})^2} = \frac{y^{-6}}{y^{-8}} = \frac{y^8}{y^6} = y^2$$

b.

$$\left(\frac{-x}{2}\right)^{-2} = \left(\frac{2}{-x}\right)^2 = \frac{2^2}{(-x)^2} = \frac{4}{x^2}$$

Question 4. (5 marks) Simplify and write the solution so that only positive exponents remain:

$$\begin{aligned}\left(\frac{x^{-3}y^2z^0}{-x^2y^{-4}z}\right)^{-3} &= \left(\frac{-x^2y^{-4}z}{x^{-3}y^2}\right)^3 \\ &= \frac{(-x^2y^{-4}z)^3}{(x^{-3}y^2)^3} \\ &= \frac{(-1)^3(x^2)^3(y^{-4})^3z^3}{(x^{-3})^3(y^2)^3} \\ &= \frac{-x^6y^{-12}z^3}{x^{-9}y^6} \\ &= \frac{-x^6x^9z^3}{y^{12}y^6} \\ &= \frac{-x^{15}z^3}{y^{18}}\end{aligned}$$

Question 5. (3 marks) Simplify:

$$\begin{aligned}x^3 + [3x - (x^3 - 3x)] - (2x - x^3) \\ = x^3 + [3x - x^3 + 3x] - 2x + x^3 \\ = x^3 + 4x\end{aligned}$$

Question 6. (2 marks) Expand and simplify:

$$\begin{aligned}2x(4x-1)(x-3) \\ = 2x[4x^2 - 12x - x + 3] \\ = 8x^3 - 26x^2 + 6x\end{aligned}$$

Question 7. (4 marks) Expand and simplify:

$$\begin{aligned}(x-2)^2 - (x+2)(x-2) + 13 \\ &= (x-2)(x-2) - [(x+2)(x-2)] + 13 \\ &= x^2 - 4x + 4 - [x^2 - 4] + 13 \\ &= -4x + 21\end{aligned}$$

Question 8. (4 marks) Divide using long division:

$$(x^3 + x - 1) \div (x + 2)$$

$$\begin{array}{r}x^2 - 2x + 5 \\ x+2 \overline{) x^3 + 0x^2 + x - 1} \\ \underline{-(x^3 + 2x^2)} \\ -2x^2 + x \\ \underline{-(-2x^2 - 4x)} \\ 5x - 1 \\ \underline{-(5x + 10)} \\ -11\end{array}$$

$$\therefore \frac{x^3 + x - 1}{x + 2} = x^2 - 2x + 5 - \frac{11}{x + 2}$$

Question 9. (2 marks) Factor completely:

$$\begin{aligned}16x^2 - 25y^2 &= 4^2x^2 - 5^2y^2 \\ &= (4x)^2 - (5y)^2 \\ &= (4x - 5y)(4x + 5y)\end{aligned}$$

Question 10. (4 marks) Simplify completely:

$$\begin{aligned}\frac{x}{x-1} - \frac{2}{x^2-1} &= \frac{x}{x-1} - \frac{2}{(x-1)(x+1)} \\ &= \frac{x(x+1)}{(x-1)(x+1)} - \frac{2}{(x-1)(x+1)} \\ &= \frac{x^2+x-2}{(x-1)(x+1)} \\ &= \frac{(x+2)\cancel{(x-1)}}{\cancel{(x-1)}(x+1)} \\ &= \frac{x+2}{x+1}\end{aligned}$$

$$\text{LCD} = (x-1)(x+1)$$

Question 11. (6 marks) Simplify completely:

$$\begin{aligned}\frac{x^2-x-2}{2x^2-8} \times \frac{18-2x^2}{x^2-5x+4} \times \frac{x^2-2x-8}{x^2-6x+9} \\ &= \frac{(x-2)(x+1)}{2(x^2-4)} \cdot \frac{2(9-x^2)}{(x-1)(x-5)} \cdot \frac{x^2-2x-8}{x^2-6x+9} \\ &= \frac{\cancel{(x-2)}(x+1)}{2\cancel{(x-2)}(x+2)} \cdot \frac{-2\cancel{(x-3)}(x+3)}{(x-1)\cancel{(x-4)}} \cdot \frac{\cancel{(x-4)}(x+2)}{(x-3)\cancel{(x-3)}} \\ &= \frac{-(x+1)(x+3)}{(x-1)(x-3)}\end{aligned}$$

Question 12. (2 marks) Solve for x:

$$-4(x-2) = 3 - (5x-1)$$

$$-4x + 8 = 3 - 5x + 1$$

$$-4x + 8 = 4 - 5x$$

$$x = -4$$

Question 13. (3 marks) Solve for x:

$$\frac{3x}{8} - \frac{1}{4} = \frac{x+5}{2}$$

$$\text{LCD} = 8$$

$$\frac{3x(8)}{8} - \frac{8 \cdot 2}{4} = \frac{(x+5)8}{2} \quad | \cdot 4$$

$$3x - 2 = 4(x+5)$$

$$3x - 2 = 4x + 20$$

$$-22 = x$$

Question 14. (5 marks) Solve for x:

$$\frac{x}{x+2} - \frac{x}{x-2} = \frac{x+20}{x^2-4}$$

$$\text{LCD} = (x-2)(x+2)$$

$$\frac{x}{x+2} - \frac{x}{x-2} = \frac{x+20}{(x-2)(x+2)}$$

$$\frac{x(x+2)(x-2) - x(x-2)(x+2)}{(x+2)(x-2)} = \frac{(x+20)(x+2)(x-2)}{(x-2)(x+2)}$$

$$x(x-2) - x(x+2) = (x+20)$$

$$x^2 - 2x - x^2 - 2x = x + 20$$

$$-4x = x + 20$$

$$-5x = 20$$

$$x = -4$$

check solution, does not make the denominator vanish. ∴ valid solution.