

Test 2

This test is graded out of 46 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formulas:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right)$$

$$h = \frac{-b}{2a} \quad k = \frac{4ac - b^2}{4a}$$

Question 1. (4 marks) Yann buys a laptop for 1700\$ and the laptop is worth 500\$ after 5 years. If ^{the} laptop's value depreciates linearly, find the function that describes the depreciation. After what period of time will the laptop be worthless?

$$(0, 1700)$$

$$(5, 500)$$

$$V = mt + b$$

$$m = \frac{500 - 1700}{5 - 0}$$

$$= \frac{-1200}{5}$$

$$= -240$$

$$\therefore V = -240t + 1700$$

$$0 = -240t + 1700$$

$$240t = 1700$$

$$t \approx 7 \text{ years}$$

\therefore the laptop is worthless after 7 years.

Question 2. Solve for x

a. (3 marks) $x^2 - 4x - 3 = 0 \Rightarrow$

b. (5 marks)

$$\frac{2}{x} + \frac{x}{x-1} = \frac{1}{x-1} \quad \text{LCD} = x(x-1)$$

$$\frac{2x(x-1)}{x} + \frac{x(x-1)}{(x-1)} = \frac{x(x-1)}{(x-1)}$$

$$2x - 2 + x^2 = x$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2 \quad x = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2}$$

$$= \frac{4 \pm \sqrt{28}}{2}$$

Verify solution:

$$x = -2 \quad x = 1$$

$$x \neq 0 \quad x - 1 \neq 0$$

$\therefore x = 1$ not a solution the only solution is $x = -2$.

Question 3. Let $f(x) = x^2 - 2x + 1$ and $g(x) = \frac{1}{2x+1}$.

a. (1 mark) Evaluate $g(3)$.

$$a) g(3) = \frac{1}{2(3)+1} = \frac{1}{7}$$

b. (2 marks) Evaluate $f(x+h)$ and simplify.

$$b) f(x+h) = (x+h)^2 - 2(x+h) + 1 \\ = x^2 + 2xh + h^2 - 2x - 2h + 1$$

c. (2 marks) Simplify $\frac{f(x+h)-f(x)}{h}$.

d. (2 marks) Evaluate $(g \circ f)(x)$.

e. (1 mark) Evaluate $(g \circ f)(-1)$.

f. (bonus 1 mark) Determine the domain of $g(x)$.

1/8

$$c) \frac{f(x+h) - f(x)}{h} = \frac{x^2 + 2xh + h^2 - 2x - 2h + 1 - [x^2 - 2x + 1]}{h} \\ = \frac{x^2 + 2xh + h^2 - 2x - 2h + 1 - x^2 + 2x - 1}{h} \\ = \frac{2xh + h^2 - 2h}{h} = \frac{h(2x + h - 2)}{h} = 2x + h - 2$$

$$d) (g \circ f)(x) = g(f(x)) \\ = g(x^2 - 2x + 1) \\ = \frac{1}{2(x^2 - 2x + 1) + 1}$$

$$e) (g \circ f)(-1) = \frac{1}{2((-1)^2 - 2(-1) + 1) + 1} = \frac{1}{2(4) + 1} = \frac{1}{9}$$

f) Range: all real numbers except $\frac{-1}{2}$: $2x+1 \neq 0$
 $2x \neq -1$
 $x \neq \frac{-1}{2}$

Question 4 The 'Clever Company' company make t-shirts with the slogan "Mathemagical!". They sell the t-shirt to the hipsters for 30\$. The fixed cost for making the t-shirts is 140\$ and 20\$ for each t-shirt (since t-shirts are fairtrade).
each a a variable cost

- a. (1 mark) Determine the revenue function, $R(x)$.
 b. (1 mark) Determine the cost function, $C(x)$.
 c. (2 marks) Determine the profit function, $P(x)$.
 d. (3 marks) Determine the break-even point and discuss its meaning.
 e. (bonus 1 mark) Determine the marginal cost and discuss its meaning.

a) $R(x) = 30x$

b) $C(x) = 20x + 140$

c) $P(x) = R(x) - C(x)$
 $= 30x - (20x + 140)$
 $= 10x - 140$

d) $0 = P(x)$
 $0 = 10x - 140$
 $140 = 10x$
 $14 = x$

∴ the break-even point is at $x=14$ (when 14 t-shirts are sold). The next t-shirt they sell, they will generate a profit.

e) $\overline{MC} = 20$, each additional shirt they produce cost an additional 20\$.

Question 5 Brittany and Giuseppe S. have determined the demand and supply function for the t-shirts sold by the 'Clever Company', while playing cards: demand: $2p + 9q = 300$
supply: $2p - q = 20$

- (4 marks) Determine the market equilibrium.
- (1 mark) Determine the q and p intercepts of the demand function.
- (1 mark) Determine the q and p intercepts of the supply function.
- (3 marks) Sketch the graph of the demand, supply function and label the market equilibrium.

① $2p + 9q = 300$

② $2p - q = 20 \Leftrightarrow q = 2p - 20$ sub into ①

→ sub into \star

$$2p + 9(2p - 20) = 300$$

$$2p + 18p - 180 = 300$$

$$20p = 480$$

$$p = 24$$

$$q = 2(24) - 20$$

$$q = 48 - 20$$

$$q = 28$$

∴ market equilibrium point at $(28, 24)$

b) $2p + 9q = 300$

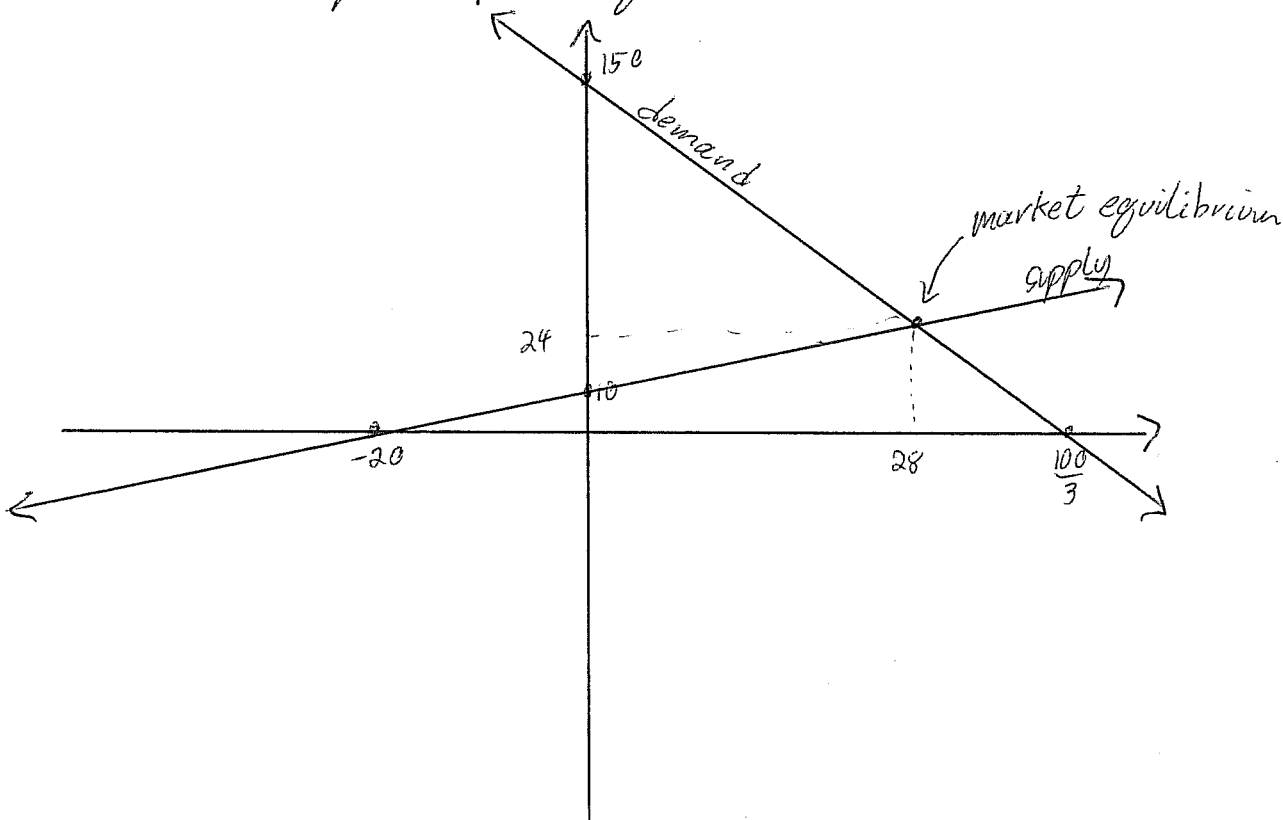
p-int: $q = 0 \Rightarrow p = 150 \therefore (0, 150)$

q-int: $p = 0 \Rightarrow q = \frac{100}{3} \therefore (\frac{100}{3}, 0)$

c) $2p - q = 20$

p-int: $q = 0 \Rightarrow p = 10 \therefore (0, 10)$

q-int: $p = 0 \Rightarrow q = -20 \therefore (-20, 0)$



Question 6. Let $f(x) = x^2 - 4x + 3$ be a quadratic function.

a. (3 marks) Determine the vertex of $f(x)$.

b. (1 mark) Determine the orientation of the parabola and state whether the vertex is a minimum or maximum.

c. (1 mark) Determine the y-intercept.

d. (3 marks) Determine the x-intercept(s).

e. (2 marks) Sketch the graph of $f(x)$.

f. (bonus 1 mark) Determine the range of $f(x)$.

b) $a=1 > 0$ \uparrow , the vertex is a minimum

c) $(0, c) = (0, 3)$ is the y-int

10

a) $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$$= \left(\frac{4}{2(1)}, f\left(\frac{4}{2(1)}\right)\right) = (2, f(2)) = (2, 2^2 - 4(2) + 3) = (2, -1)$$

d) $0 = f(x)$

$$0 = x^2 - 4x + 3$$

$$0 = (x-3)(x-1)$$

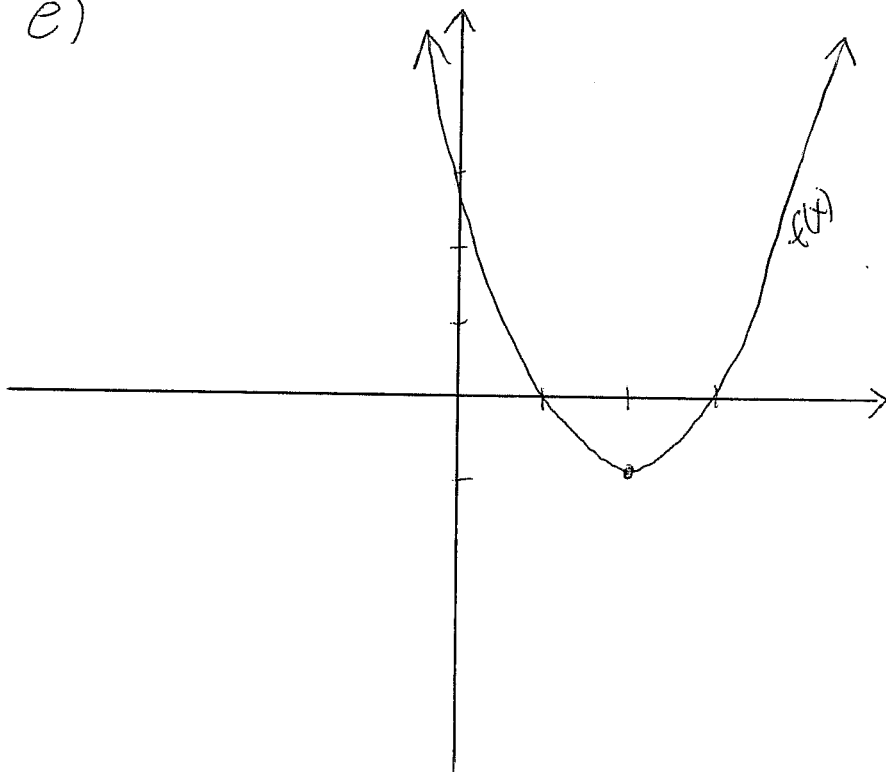
$$x-3=0$$

$$x=3$$

$$x-1=0$$

$$x=1$$

e)



f) Range: $[-1, \infty)$