Dawson College: Business Mathematics: 201-914-DW-S02: Fall 2008

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Test 3

This test is graded out of 46 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formulas:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \qquad \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \qquad h = \frac{-b}{2a} \quad k = \frac{4ac-b^2}{4a}$$

$$I = Prt \qquad S = P + I = P(1+rt)$$

$$S = Pe^{rt} \qquad FV = PV\left(1+\frac{j}{m}\right)^{mt}$$

Question 1. *(4 marks)* Express the logarithm as the sum and difference of logarithms (with no powers on $(x+1)$, $(x+2)$ and $(x+3)$).

$$\log\left[\frac{(x+1)^3(x+2)^4}{(x+3)^2}\right]$$

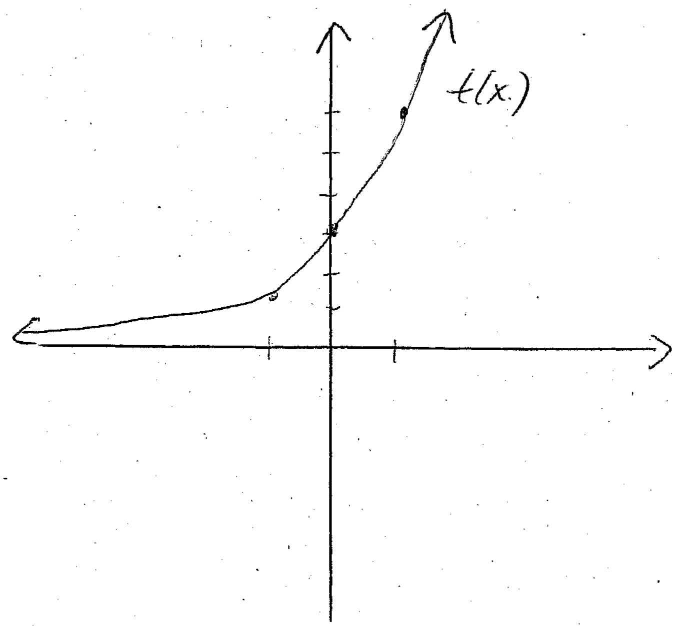
$$= \log\left[(x+1)^3(x+2)^4\right] - \log(x+3)^2$$

$$= \log(x+1)^3 + \log(x+2)^4 - \log(x+3)^2$$

$$= 3\log(x+1) + 4\log(x+2) - 2\log(x+3)$$

Question 2. *(4 marks)* Sketch a graph of $f(x) = 3(2^x)$.

X	f(x)
-1	$f(-1) = 3(2^{-1}) = \frac{3}{2}$
0	$f(0) = 3(2^0) = 3$
1	$f(1) = 3(2^1) = 6$



Question 3. Alex invests \$900 in a simple interest scheme at a rate of 4.25% per year for 7 months.

a. *(2 marks)* How much interest did Alex gain?

b. *(2 marks)* What is the future value of Alex's investment?

a) $I = Prt = 900(0.0425)\left(\frac{7}{12}\right) \doteq \$22.31$

b) $S = P + I = 900 + 22.31 = \$922.31$

Question 4. *(4 marks)* Let $p = 2q^2 + 100q + 3600$ be the supply function for a product and $p = 500q - 2q^2$ be the demand function, find the market equilibrium.

$$2q^2 + 100q + 3600 = 500q - 2q^2$$

$$4q^2 - 400q + 3600 = 0$$

$$q^2 - 100q + 900 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-100) \pm \sqrt{(-100)^2 - 4(1)(900)}}{2(1)}$$

$$= \frac{100 \pm \sqrt{10000 - 3600}}{2}$$

$$= \frac{100 \pm 80}{2}$$

$$= 90 \quad \text{or} \quad 10$$

$p = 500(90) - 2(90)^2$
$= 28\,800$

$p = 500(10) - 2(10)^2$
$= 4800$

Question 5. Let $p = -3x + 200$ be the price of a product, where p is the price x items are sold.

a. *(2 marks)* Find the revenue function.

b. *(4 marks)* Find the number of items sold that maximize the revenue function.

a) $R(x) = px = (-3x+200)x = -3x^2 + 200x$

b) a quadratic function is maximized or minimized at the vertex.

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) = \left(\frac{-200}{2(-3)}, f(33.\bar{3})\right)$$
$$= (33.\bar{3}, f(33.\bar{3}))$$

$\therefore$ The revenue is maximized at about 33 items.

Question 6. *(4 marks)* What interest will be earned if $9 000 is invested for 26 months at 6% compounded monthly.

$FV = PV(1+i)^n$ $\qquad m = 12$

$= 9000(1+0.005)^{26}$ $\qquad i = \frac{j}{m} = \frac{6\%}{12} = 0.005$

$= \$10246.19$ $\qquad n = mt = \cancel{12}\left(\frac{26}{\cancel{12}}\right) = 26$

$FV = PV + I$

$10246.19 = 9000 + I$

$I = 10\,246.19 - 9000$

$= \$1\,246.19$

Question 7. *(4 marks)* How long *(in years)* would $5 000 have to be invested at 3%, compounded continuously, to amount to $11 000.

$$S = Pe^{rt}$$

$$11000 = 5000e^{0.03t}$$

$$2.2 = e^{0.03t}$$

$$\ln 2.2 = \ln e^{0.03t}$$

$$\ln 2.2 = 0.03t$$

$$\frac{\ln 2.2}{0.03} = t$$

$$26 \text{ years} \doteq t$$

Question 8. *(4 marks)* A sum of $25 000 would have to be invested at what nominal interest rate, coumpounded quartely, to amount to $30 000 in 10 years.

$$FV = PV(1+i)^n$$

$$m = 4$$

$$i = \frac{j}{m} = \frac{j}{4}$$

$$n = mt = 4(10) = 40$$

$$30000 = 25000\left(1+\frac{j}{4}\right)^{40}$$

$$1.2 = \left(1+\frac{j}{4}\right)^{40}$$

$$1.2^{\frac{1}{40}} = 1+\frac{j}{4}$$

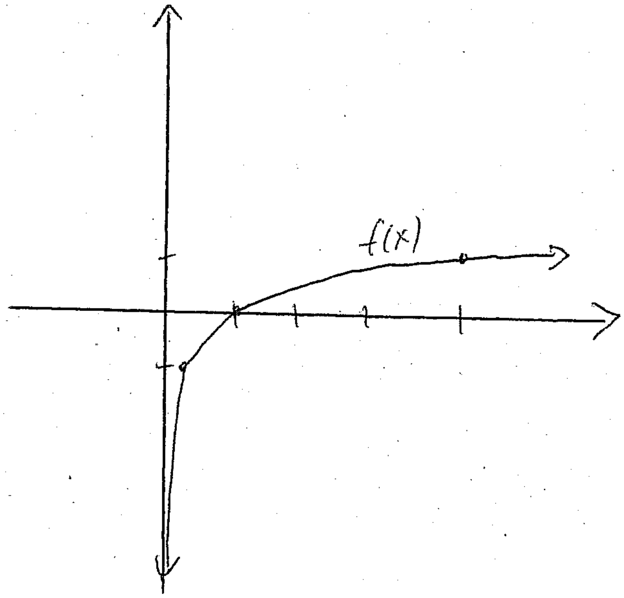
$$1.2^{\frac{1}{40}} - 1 = \frac{j}{4}$$

$$4\left(1.2^{\frac{1}{40}}-1\right) = j$$

$$1.8\% = j$$

Question 9. *(4 marks)* Sketch the graph of $f(x) = \log_4(x)$.

x	$f(x)$
$\frac{1}{4}$	$f(\frac{1}{4}) = \log_4(\frac{1}{4}) = -1$
1	$f(1) = \log_4 1 = 0$
4	$f(4) = \log_4 4 = 1$



Question 10. Evaluate

a. *(2 marks)* $\log_4 16 = 2$

b. *(2 marks)* $\log_9 \frac{1}{9} = -1$

Question 11. *(4 marks)* What amount needs to be invested in order to have \$8 500 in 265 days at a rate of 9.5% p.a.

$$S = P(1+rt)$$

$$P = \frac{S}{(1+rt)}$$

$$= \frac{8500}{\left(1+0.095\left(\frac{265}{365}\right)\right)}$$

$$= \$7\,951.56$$