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Test 2

This test is graded out of 46 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formulas:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \qquad h = \frac{-b}{2a} \quad k = \frac{4ac - b^2}{4a}$$

Question 1. Let $f(x) = x^2 - 3x + 2$ and $g(x) = \frac{1}{\sqrt{x+1}}$.

  a. *(1 mark)* Evaluate $g(3)$.

  b. *(2 marks)* Evaluate $f(x+h)$ and simplify.

  c. *(2 marks)* Simplify $\frac{f(x+h)-f(x)}{h}$.

  d. *(2 marks)* Evaluate $(g \circ f)(x)$.

  e. *(1 mark)* Evaluate $(g \circ f)(2)$.

  f. *(bonus 1 mark)* Determine the domain of $g(x)$.

a) $g(3) = \dfrac{1}{\sqrt{3+1}}$

$= \dfrac{1}{\sqrt{4}}$

$= \dfrac{1}{2}$

b) $f(x+h) = (x+h)^2 - 3(x+h) + 2$

$\qquad\qquad = x^2 + 2xh + h^2 - 3x - 3h + 2$

c) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{x^2 + 2xh + h^2 - 3x - 3h + 2 - [x^2 - 3x + 2]}{h}$

$\qquad\qquad = \dfrac{x^2 + 2xh + h^2 - 3x - 3h + 2 - x^2 + 3x - 2}{h}$

$\qquad\qquad = \dfrac{2xh + h^2 - 3h}{h}$

$\qquad\qquad = \dfrac{h(2x + h - 3)}{h} = 2x + h - 3$

d) $(g \circ f)(x) = g(f(x))$

$\qquad\qquad = g(x^2 - 3x + 2)$

$\qquad\qquad = \dfrac{1}{\sqrt{x^2 - 3x + 2 + 1}} = \dfrac{1}{\sqrt{x^2 - 3x + 3}}$

f) Range: $x > -1$

e) $(g \circ f)(2) = \dfrac{1}{\sqrt{2^2 - 3(2) + 3}} = \dfrac{1}{\sqrt{1}} = 1$

Question 2. Let $f(x) = x^2 + 6x + 5$ be a quadratic function.

a. *(3 marks)* Determine the vertex.

b. *(1 mark)* Determine the orientation and state whether the vertex is a minimum or maximum.

c. *(1 mark)* Determine the y-intercept.

d. *(3 marks)* Determine the x-intercept(s).
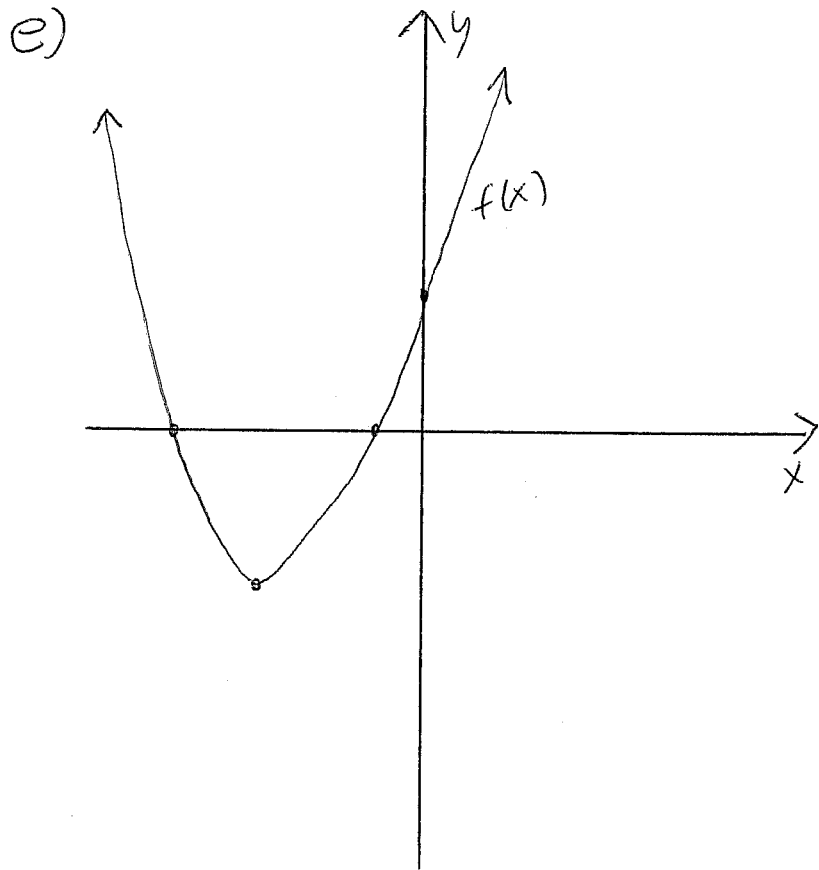
e. *(2 marks)* Sketch the graph of $f(x)$.

f. *(bonus 1 mark)* Determine the range of $f(x)$.

c) $y\text{-int}: (0,c) = (0,5)$

b) orientation: $a > 1$

∴ vertex a minimum

a) $\left(\dfrac{-b}{2a}, f\left(\dfrac{-b}{2a}\right)\right) = \left(\dfrac{-6}{2(1)}, f\left(\dfrac{-6}{2(1)}\right)\right)$

$= (-3, f(-3))$

$= (-3, (-3)^2 + 6(-3) + 5)$

$= (-3, -4)$

d) $0 = f(x)$

$0 = x^2 + 6x + 5$

$0 = (x+1)(x+5)$

$x + 1 = 0 \qquad x + 5 = 0$

$x = -1 \qquad x = -5$

e)

f) Range: $[-4, \infty$

Question 3. The 'Clever Company' company make t-shirts with the slogan "Mathemagical". They sell the t-shirt to the hipsters for 25$. The fixed cost for making the t-shirts is 200$ and 15$ for each t-shirt *(since they are fairtrade)*.

each   *λ*
                                                                        Variable cost

a. *(1 mark)* Determine the revenue function, $R(x)$.

b. *(1 mark)* Determine the cost function, $C(x)$.

c. *(2 marks)* Determine the profit function, $P(x)$.

d. *(3 marks)* Determine the break-even point and discuss its meaning.

e. *(bonus 1 mark)* Determine the marginal profit and discuss its meaning.

a) $R(x) = 25x$

b) $C(x) = 15x + 200$

c) $P(x) = R(x) - C(x)$

$\qquad = 25x - (15x + 200)$

$\qquad = 10x - 200$

d) $0 = P(x)$

$\quad 0 = 10x - 200$

$\quad 200 = 10x$

$\quad 20 = x$

After the Clever Company sell 20 t-shirts it will generate profit.

e) $\overline{MP} = 10$

Each additional t-shirts sold the Clever Company will make an additional 10$ profit.

Question 4. Brittany and Giuseppe have determined the demand and supply function for the t-shirt of the 'Clever Company', while playing cards: demand: $2p + q = 240$
supply: $2p - 9q = 10$

    a. *(4 marks)* Determine the market equilibrium.

    b. *(1 mark)* Determine the q and p intercepts of the demand function.

    c. *(1 mark)* Determine the q and p intercepts of the supply function.

    d. *(3 marks)* Sketch the graph of the demand, supply function and label the market equilibrium.

a) ① $2p + q = 240$ $\iff$ $q = -2p + 240$ ✱

② $2p - 9q = 10$

sub into ②

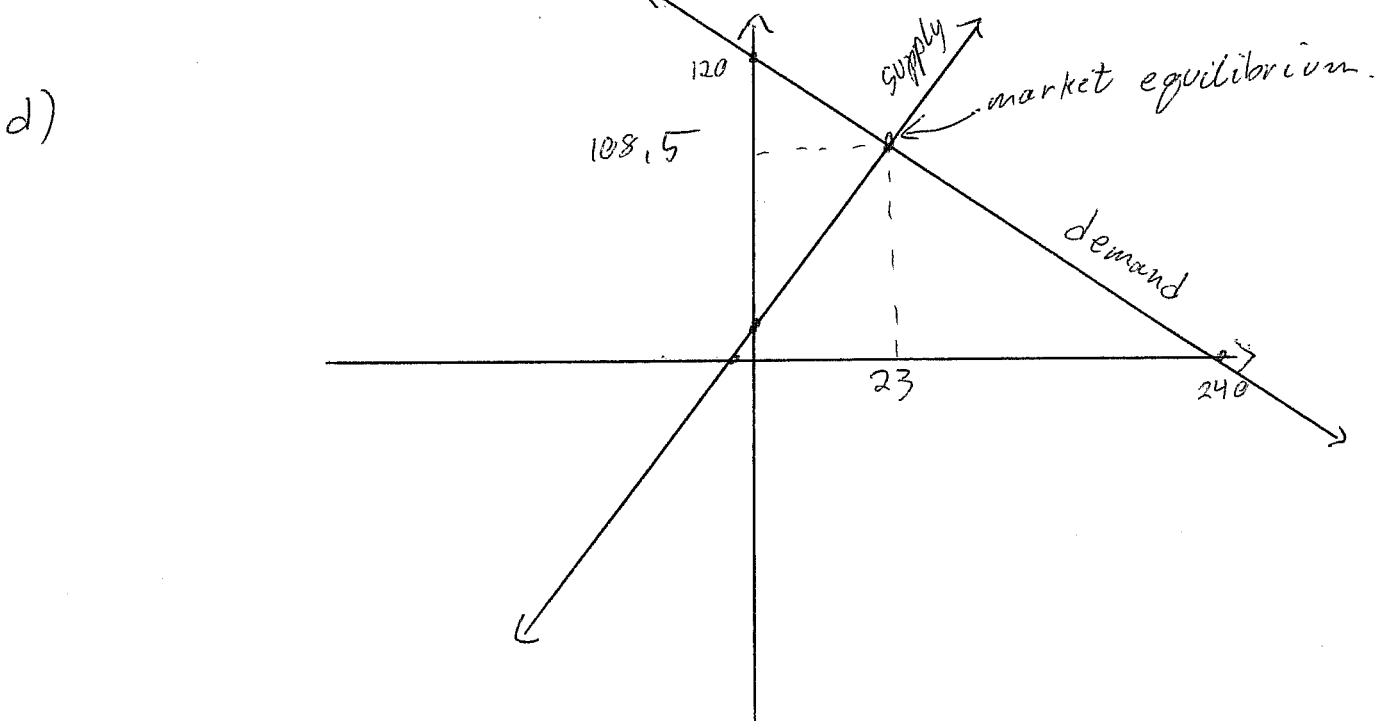$$2p - 9(-2p + 240) = 10$$
$$2p + 18p - 2160 = 10$$
$$20p = 2170$$
$$p = 108.5$$

sub into ✱

$$q = -2(108.5) + 240$$
$$q = 23$$

∴ the market equilibrium is $(23, 108.5)$

b) demand: $2p + q = 240$

q-int: $p = 0 \Rightarrow q = 240$ ∴ $(240, 0)$

p-int: $q = 0 \Rightarrow p = 120$ ∴ $(0, 120)$

c) supply: $2p - 9q = 10$

q-int: $p = 0 \Rightarrow q = -\frac{10}{9}$ ∴ $(-\frac{10}{9}, 0)$

p-int: $q = 0 \Rightarrow p = 5$ ∴ $(0, 5)$

d)

Question 5. *(4 marks)* Yann buys a laptop for 1700$ and the laptop is worthless after 5 years. If the laptop's value depreciates linearly, find the function that describes the depreciation. After what period of time will the laptop be worth 1000$?

$(0, 1700)$

$(5, 0)$

$V = mt + b$

$m = \dfrac{0 - 1700}{5 - 0}$

$= -340$

$V = -340t + b$

$V = -340t + 1700$

$1000 = -340t + 1700$

$-700 = -340t$

$2 \overset{\circ}{=} t$

· It will be worth 1000 after about 2 years

Question 6. Solve for x

a. *(3 marks)* $x^2 - 2x - 2 = 0$ $\Longrightarrow$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

b. *(5 marks)*

LCD
$= x(x-4)$

$1 = \dfrac{35}{x-4} - \dfrac{35}{x}$

$= \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$

$x(x-4) = \dfrac{35 \cdot x(x-4)}{x-4} - \dfrac{35 \cdot x(x-4)}{x}$

$= \dfrac{2 \pm \sqrt{4+8}}{2}$

$= \dfrac{2 \pm \sqrt{12}}{2}$

$x^2 - 4x = 35x - 35x + 140$

$0 = x^2 - 4x - 140$

$0 = (x - 14)(x + 10)$

$X - 14 = 0 \qquad X + 10 = 0$

$X = 14 \qquad X = -10$

Verify solutions:

$X = 14 \qquad X = -10$

$x : \neq 0 \qquad \neq 0$

$x - 4 : \neq 0 \qquad \neq 0$

solutions are
$x = 14$ and $x = -10$.