Name: _____
Student ID: _____

Test 3

This test is graded out of 47 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formulas:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \qquad \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \qquad h = \frac{-b}{2a} \quad k = \frac{4ac-b^2}{4a}$$

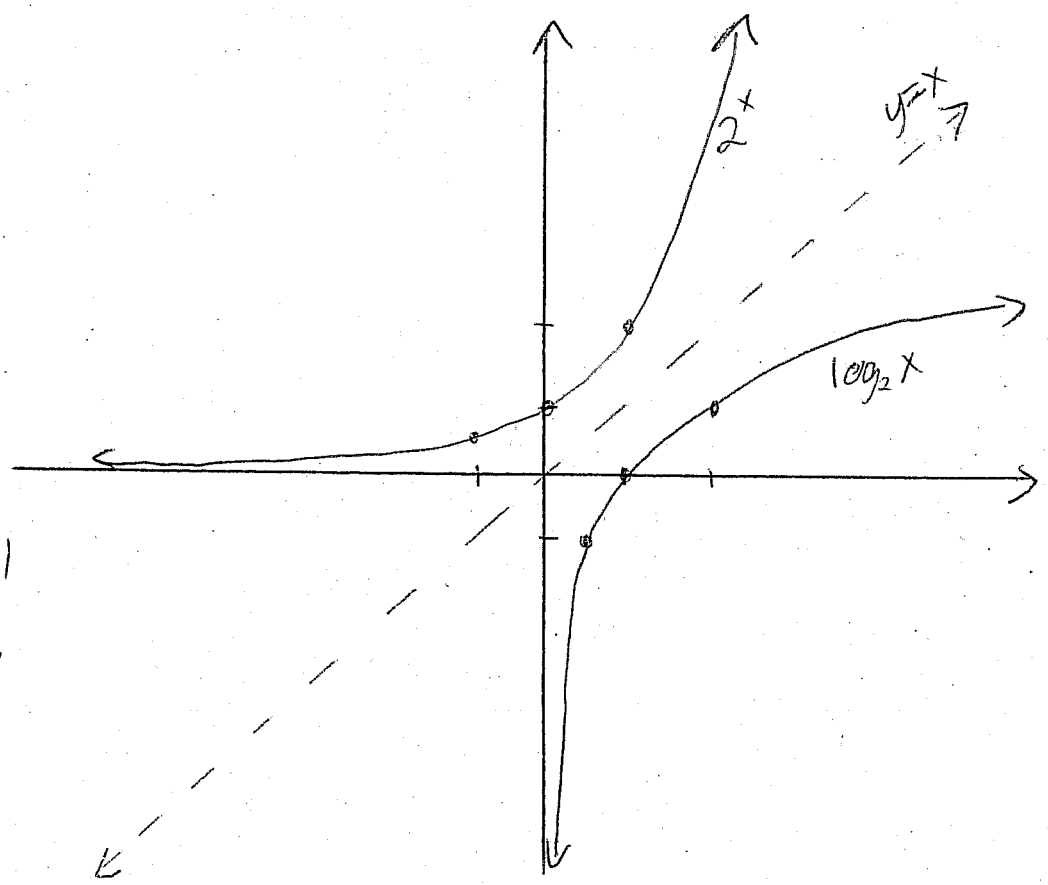
$$I = Prt \qquad S = P + I = P(1+rt)$$

$$S = Pe^{rt} \qquad FV = PV\left(1+\frac{j}{m}\right)^{mt}$$

Question 1. *(9 marks)* Sketch the graph of $f(x) = 2^x$, $g(x) = \log_2(x)$ and $y = x$ on the same cartesian plane.

x	2^x
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$

x	$\log_2 x$
$\frac{1}{2}$	$\log_2 \frac{1}{2} = -1$
1	$\log_2 1 = 0$
2	$\log_2 2 = 1$



Question 2.

a. *(4 marks)* Express the logarithms as a single logarithm with a coefficient of one.

$$2\log(x+1)+\frac{1}{2}\log(x+2)-3\log(x+3)$$

b. *(2 marks)* $\log_4 2$

c. *(3 marks)* Solve for x.

$$\log(2x-4)=2$$

a) $\log(x+1)^2 + \log\sqrt{x+2} - \log(x+3)^3$

$= \log \dfrac{(x+1)^2\sqrt{x+2}}{(x+3)^3}$

b) $\log_4 2 = \frac{1}{2}$

c) $\log(2x-4) = 2$

$2x-4 = 10^2$

$2x = 10^2+4$

$2x = 104$

$x = 52$

Question 3. John loans $900 for 100 days to Emma at a rate of 1.25% per year.

a. *(2 marks)* How much interest does Emma owe John?

b. *(2 marks)* What is the future value of the loan?

a) $I = Prt$

$= 900(0.0125)\left(\frac{100}{365}\right)$

$= \$3.08$

b) $S = P + I$

$= 900 + 3.08$

$= \$903.80$

Question 4. *(4 marks)* What interest will be earned if $9 000 is invested for 19 months at 6% compounded continuously.

$S = Pe^{rt}$

$S = 9000\, e^{0.06\left(\frac{19}{12}\right)}$

$S = \$9896.93$

$I = S - P = 9896.93 - 9000$

$= \$896.93$

Question 5. Let $C(x) = 2x^2 + 100x + 3600$ be the cost function and $R(x) = 500x - 2x^2$ be the revenue function.

a. *(1 mark)* Find the profit function, $P(x)$.

b. *(4 marks)* Find the break-even point.

c. *(4 marks)* Find the number of items sold that maximize the profit function and state the maximum profit.

a) $P(x) = R(x) - C(x)$

$= 500x - 2x^2 - (2x^2 + 100x + 3600)$

$= 400x - 4x^2 - 3600$

b) $0 = P(x)$

$0 = 400x - 4x^2 - 3600$

$0 = x^2 - 100x + 900$

$0 = (x-10)(x-90)$

$x = 10$ $\quad$ $x = 90$

∴ two break-even point.

c) Profit is maximized at the vertex

$\left(\frac{-b}{2a}, P\left(\frac{-b}{2a}\right)\right) = \left(\frac{-400}{2(4)}, P\left(\frac{-400}{2(4)}\right)\right)$

$= (50, P(50))$

$= (50, 6400)$

∴ the profit is maximized at 50 and the maximum profit is 6400.

Question 6. *(4 marks)* How long *(in years)* would \$6 000 have to be invested at 12%, compounded quartely, to amount to \$35400.

$$FV = PV\left(1 + \frac{j}{m}\right)^{mt} \qquad m = 4, \; j = 12\%$$

$$35400 = 6000\left(1 + \frac{0.12}{4}\right)^{4t}$$

$$5.9 = \left(1 + \frac{0.12}{4}\right)^{4t}$$

$$5.9 = (1.03)^{4t}$$

$$\ln 5.9 = \ln (1.03)^{4t}$$

$$\ln 5.9 = 4t \ln (1.03)$$

$$t = \frac{\ln 5.9}{4 \ln 1.03} = 15 \text{ years.}$$

Question 7. *(4 marks)* A sum of \$25 000 would have to be invested at what interest rate to amount to \$30 000 in 200 days.

$$S = P(1 + rt)$$

$$30000 = 25000\left(1 + r\left(\frac{200}{365}\right)\right)$$

$$30000 = 25000 + 25000\left(\frac{40}{73}\right)r$$

$$5000 = 25000\left(\frac{40}{73}\right)r$$

$$r = 0.365$$

$$= 36.5\%$$

Question 8. *(4 marks)* What amount needs to be invested in order to have \$6 200 in 199 days at a rate of 4.5% compounded daily.

$$FV = PV\left(1 + \frac{j}{m}\right)^{mt}$$

$$6200 = PV\left(1 + \frac{0.045}{365}\right)^{365\left(\frac{199}{365}\right)}$$

$$6200 = PV\left(1 + \frac{0.045}{365}\right)^{199}$$

$$PV = \frac{6200}{\left(1 + \frac{0.045}{365}\right)^{199}}$$

$$= \$6049.75$$