Name: _____
Student ID: _____

Test 3

This test is graded out of 47 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formulas:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \qquad \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \qquad h = \frac{-b}{2a} \quad k = \frac{4ac-b^2}{4a}$$

$$I = Prt \qquad S = P + I = P(1+rt)$$

$$S = Pe^{rt} \qquad FV = PV\left(1+\frac{j}{m}\right)^{mt}$$

Question 1. Emma loans \$1100 for 200 days to Ba Jin at a rate of 2.25% per year.

a. *(2 marks)* How much interest does Ba Jin owe Emma?

b. *(2 marks)* What is the future value of the loan?

a) $I = Prt$

$= (1100)(0.0225)\left(\frac{200}{365}\right)$

$= \$13.56$

b) $S = P + I$

$= 1100 + 13.56$

$= \$1113.56$

Question 2. *(4 marks)* What interest will be earned if \$9 000 is invested for 255 days at 4% compounded continuously.

$S = Pe^{rt}$

$= 9000e^{0.04\left(\frac{255}{365}\right)}$

$= \$9255.05$

$S = P + I$

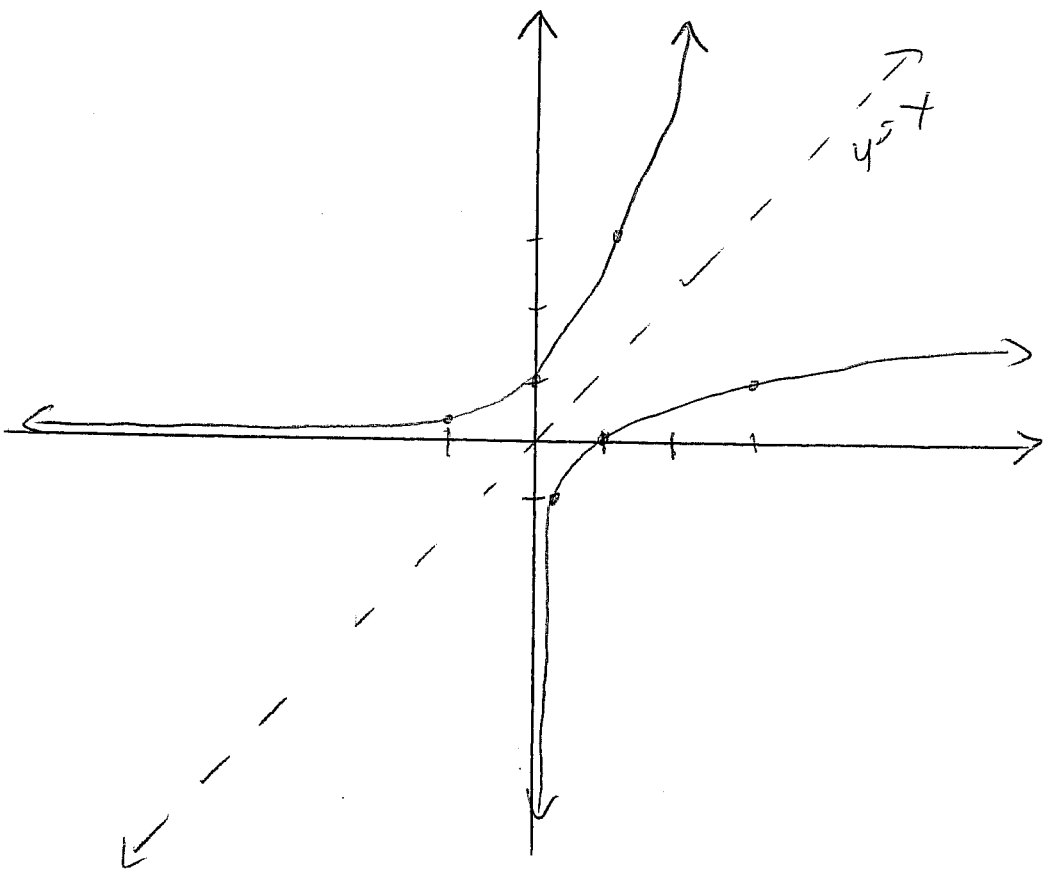
$9255.05 = 9000 + I$

$I = \$255.05$

Question 3. *(9 marks)* Sketch the graph of $f(x) = 3^x$, $g(x) = \log_3(x)$ and $y = x$ on the same cartesian plane.

x	3^x
-1	$3^{-1} = \frac{1}{3}$
0	$3^0 = 1$
1	$3^1 = 3$

x	$\log_3 x$
$\frac{1}{3}$	$\log_3 \frac{1}{3} = -1$
1	$\log_3 1 = 0$
3	$\log_3 3 = 1$



Question 4. *(4 marks)* How long *(in years)* would \$16 000 have to be invested at 6%, compounded monthly, to amount to \$55 400.

$$FV = PV\left(1 + \frac{j}{m}\right)^{mt}$$

$$55400 = 16000\left(1 + \frac{0.06}{12}\right)^{12t}$$

$$3.4625 = (1.005)^{12t}$$

$$\ln 3.4625 = \ln (1.005)^{12t}$$

$$\ln 3.4625 = 12t \ln(1.005)$$

$$t = \frac{\ln 3.4625}{12 \ln(1.005)}$$

$$= 21 \text{ years}$$

Question 5. *(4 marks)* A sum of \$5 000 would have to be invested at what interest rate to amount to \$6000 in 9 months.

$$S = P(1 + rt)$$

$$6000 = 5000\left(1 + r\left(\frac{9}{12}\right)\right)$$

$$6000 = 5000 + 3750r$$

$$1000 = 3750r$$

$$r = 0.2\bar{6}$$

$$\doteq 26\%$$

Question 6.

a. *(4 marks)* Express the logarithms as a single logarithm with a coefficient of one.

$$\frac{1}{2}\log(x+1)+2\log(x+3)-3\log(x+2)$$

b. *(2 marks)* $\log_9 3$

c. *(3 marks)* Solve for x.

$$\log(4+4x)=3$$

a) $\log(x+1)^{\frac{1}{2}} + \log(x+3)^2 - \log(x+2)^3$

$= \log \dfrac{(x+1)^{\frac{1}{2}}(x+3)^2}{(x+2)^3}$

b) $\log_9 3 = \frac{1}{2}$

c) $\log(4+4x) = 3$

$4+4x = 10^3$

$+4x = 1000 - 4$

$x = 249$

Question 7. Let $C(x) = 3200 + 1500x$ be the cost function and $R(x) = 1700x - 2x^2$ be the revenue function.

a. *(1 mark)* Find the profit function, $P(x)$.

b. *(4 marks)* Find the break-even point.

c. *(4 marks)* Find the number of items sold that maximize the profit function and find the maximum profit.

a)

$$P(x) = R(x) - C(x)$$
$$= 1700x - 2x^2 - (3200 + 1500x)$$
$$= 200x - 2x^2 - 3200$$

b)

$$0 = P(x)$$
$$0 = 200x - 2x^2 - 3200$$
$$0 = 100x - x^2 - 1600$$
$$0 = x^2 - 100x + 1600$$
$$0 = (x-20)(x-80)$$

$x = 20$ $\qquad$ $x = 80$

c) The profit function is maximized at the vertex.

$$\left(\frac{-b}{2a}, P\left(\frac{-b}{2a}\right)\right) = \left(\frac{-200}{2(-2)}, P\left(\frac{-200}{2(-2)}\right)\right)$$
$$= (50, P(50))$$
$$= (50, 200(50) - 2(50)^2 - 3200)$$
$$= (50, 1600)$$

∴ profit maximized at $x = 50$ and max profit at 1800.

Question 8. *(4 marks)* What amount needs to be invested in order to have $2 200 in 99 days at a rate of 5.4% compounded daily.

$$FV = PV\left(1 + \frac{j}{m}\right)^{mt}$$

$$2200 = PV\left(1 + \frac{0.054}{365}\right)^{365\left(\frac{99}{365}\right)}$$

$$PV = \frac{2200}{\left(1 + \frac{0.054}{365}\right)^{99}}$$

$$= \$2168.01$$