SOLUTIONS - Extra Related Rates problems

(Q1) Given $\frac{dV}{dt} = 1 cm^3/s$

Looking for dd/dt when $d = 10$ cm

(d = diameter of sphere)

$V = \frac{4}{3}\pi r^3$ but $r = d/2$ $\left(\text{radius} = \frac{\text{diameter}}{2}\right)$

so $V = \frac{4}{3}\pi (d/2)^3$

$V = \frac{4}{24}\pi d^3$

$V = \frac{\pi}{6} d^3$

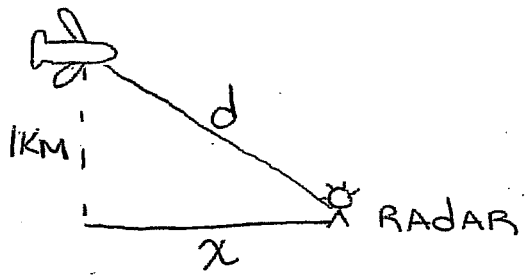
(differentiate with respect to t)

$\frac{dV}{dt} = \frac{\pi}{2} d^2 \frac{dd}{dt}$

$1 = \frac{\pi}{2}(10)^2 \frac{dd}{dt}$

$\boxed{\frac{dd}{dt} = 1/50\pi \text{ cm/s}}$.

(Q2)



d = distance between radar station & plane

$\frac{dx}{dt} = 500 \text{ Km/h}$

Looking for $\frac{dd}{dt}$ when $d = 2$ Km

Using pythagorean theorem

$$1^2 + x^2 = d^2$$

differentiate with respect to t

$$2x \frac{dx}{dt} = 2d \frac{dd}{dt}$$

Find x when $d = 2$

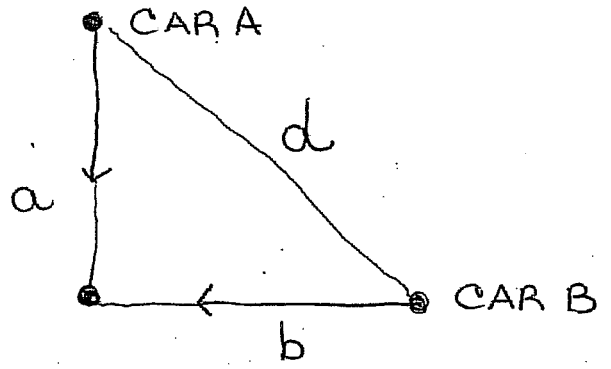
$$1^2 + x^2 = 2^2$$
$$x^2 = 3$$
$$x = \sqrt{3}$$

$$2\sqrt{3}(500) = 2(2) \frac{dd}{dt}$$

$$\frac{dd}{dt} = \frac{1000\sqrt{3}}{4}$$

$$\boxed{\frac{dd}{dt} = 250\sqrt{3} \text{ Km/h}}$$

(Q3)



CAR a : $da/dt = 60\ Km/h$

$db/dt = 25\ Km/h$

Looking for dd/dt 2 HOURS LATER

2 HOURS LATES

CAR A HAS TRAVELED 120 Km

CAR B HAS TRAVELED 50 Km

So $a = 120$ & $b = 50$ & $a^2 + b^2 = d^2$

$120^2 + 50^2 = d^2$

$d = 130$

$$a^2 + b^2 = d^2$$

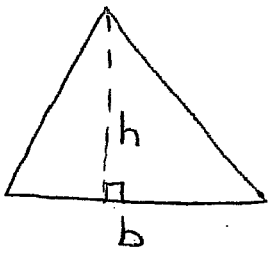
$$2a\, da/dt + 2b\, db/dt = 2d\, dd/dt$$

$$2(120)(60) + 2(50)25 = 2(130)\, dd/dt$$

$$\frac{14400 + 2500}{260} = dd/dt$$

$$\boxed{dd/dt = 65\ Km/h}$$

(Q4)



$dh/dt = 1 \text{ cm/min}$

$dA/dt = 2 \text{ cm}^2/\text{min}$

Looking for db/dt when $h=10$ & $A=100$

$$A = \frac{1}{2} b h$$

$$\frac{dA}{dt} = \frac{1}{2}\frac{db}{dt} h + \frac{1}{2}\frac{dh}{dt} b \quad \text{(PRODUCT RULE)}$$

WHEN $A=100$ & $h=10$

$$100 = \frac{1}{2} b (10)$$

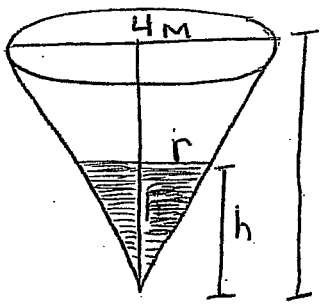
$$b = 20$$

$$2 = \frac{1}{2}\frac{db}{dt}(10) + \frac{1}{2}(1)(20)$$

$$-8 = 5\frac{db}{dt}$$

$$\boxed{\frac{db}{dt} = -\frac{8}{5} \text{ cm/min}}$$

(Q5)



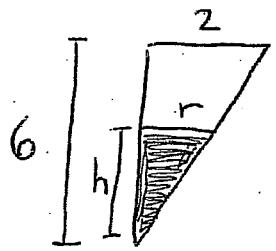
$\frac{dh}{dt} = 0.2$ WHEN $h = 2M$

Looking for dV/dt

V = volume of water in tank

$$V = \frac{1}{3}\pi r^2 h$$

Express r in terms of h



$$\frac{r}{h} = \frac{2}{6}$$

$$r = h/3$$

$$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$$

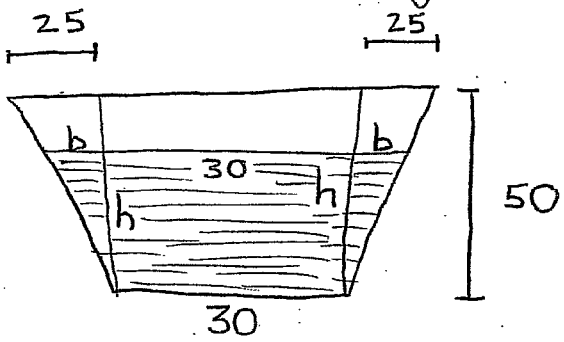
$$V = \frac{\pi}{27} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{9} h^2 \, dh/dt$$

$$\frac{dv}{dt} = \frac{\pi}{9}(2)^2 (0.2)$$

$$\boxed{\frac{dV}{dt} = \frac{0.8}{9} \, m^3/MIN}$$

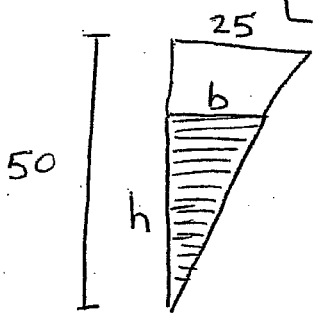
(Q6) Looking for dh/dt when $h = 30$



Given $\frac{dV}{dt} = 20$

V = (Area of trapezoid) × length of trough

$$= \left[30h + 2\left(\frac{1}{2}bh\right)\right] \times 1000$$



$$\frac{b}{h} = \frac{25}{50}$$

$$b = \frac{h}{2}$$

$$V = \left[30h + 2\left(\frac{1}{2}\left(\frac{h}{2}\right)h\right)\right] \times 1000$$

$$V = 30000h + 500h^2$$

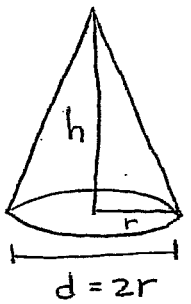
$$\frac{dV}{dt} = 30000\,dh/dt + 1000h\,dh/dt$$

$$20 = (30000 + 1000(30))\,dh/dt$$

$$20 = 60000\,dh/dt$$

$$\boxed{dh/dt = \frac{1}{3000} \text{ cm/min}}$$

(Q7)



Looking for dh/dt when $h = 10$

Given $\frac{dV}{dt} = 30$

& $d = h$

$2r = h$

$r = h/2$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$$

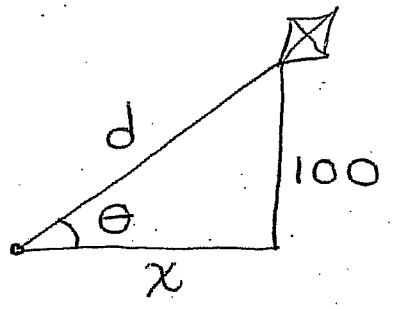
$$V = \frac{\pi}{12} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$30 = \frac{\pi}{4} (10)^2 \frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{120}{100\pi} = \frac{6}{5\pi} \text{ ft/s}}$$

(Q8)



Looking for $d\theta/dt$ when $d = 200$

When $d = 200$

$$x^2 + 100^2 = 200^2$$
$$x^2 = 200^2 - 100^2$$
$$x^2 = 40000 - 10000$$
$$x^2 = 30000$$
$$x = \sqrt{30000}$$
$$= 100\sqrt{3}$$

Given $dx/dt = 200$

$$\tan\theta = \frac{100}{x}$$

$$\sec^2\theta \, d\theta/dt = 100(-x^{-2}) \, dx/dt$$

When $d = 200$ & $x = 100\sqrt{3}$

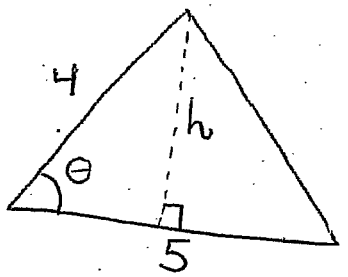
$$\cos\theta = \frac{100\sqrt{3}}{200} = \frac{\sqrt{3}}{2}$$

$$\sec\theta = \frac{2}{\sqrt{3}}$$

$$\left(\frac{2}{\sqrt{3}}\right)^2 \frac{d\theta}{dt} = \frac{-100}{(100\sqrt{3})^2} \quad (8)$$

$$\frac{d\theta}{dt} = \frac{-8}{300}\left(\frac{3}{4}\right) = \frac{-1}{} \text{ ft/}$$

(Q9)



Looking for $\frac{dA}{dt}$ when $\theta = \pi/3$

Given $d\theta/dt = 0.06$

$$A = \frac{bh}{2} \qquad A = \frac{5h}{2}$$

We want to express AREA in terms of the VARIABLE θ

$$\sin\theta = \frac{h}{4}$$

$$h = 4\sin\theta$$

$$A = \frac{5(4\sin\theta)}{2}$$

$$A = 10\sin\theta$$

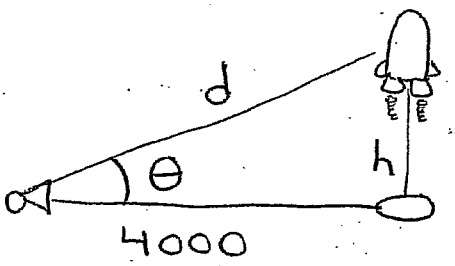
$$dA/dt = 10\cos\theta \, d\theta/dt$$

$$dA/dt = 10\cos(\pi/3)\ 0.06$$

$$dA/dt = 10\left(\frac{1}{2}\right) 0.06$$

$$\boxed{dA/dt = 0.3\ m^2/s}$$

(Q10)



(a) Looking for dd/dt when $h = 3000$

Given $dh/dt = 600$

$$(4000)^2 + h^2 = d^2$$

$$2h\, dh/dt = 2d\, dd/dt$$

When $h = 3000$

$$(4000)^2 + (3000)^2 = d^2$$

$$d = 5000$$

$$2(3000)(600) = 2(5000)\, dd/dt$$

$$\boxed{dd/dt = 360 \text{ ft/s}}$$

(b) Looking for $d\theta/dt$ when $h = 3000$

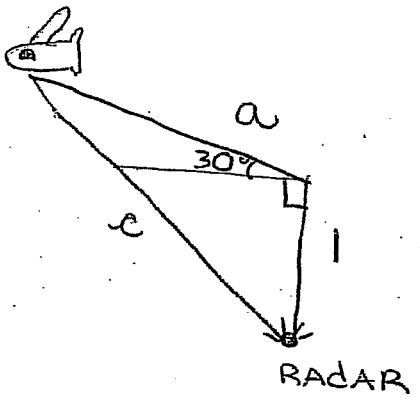
$$\tan\theta = \frac{h}{4000}$$

$$\sec^2\theta\, d\theta/dt = \frac{1}{4000}\, dh/dt$$

$$\cos\theta = \frac{4000}{5000} = 4/5 \quad \Big\} \quad (5/4)^2\, d\theta/dt = \frac{1}{4000}(600)$$

$$\boxed{d\theta/dt = 0.12 \text{ rad/s}}$$

(Q11)



Looking for dc/dt 1 min later

After 1 min.

$$a = 300/60$$
$$= 5 \text{ km}$$

Given $da/dt = 300$

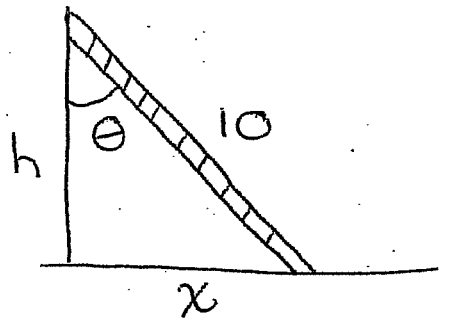
Using the Law of Cosines we can find c one minute later.

$$c^2 = a^2 + 1^2 - 2a\cos(120°)$$
$$c^2 = (5)^2 + 1^2 - 2(5)(-1/2)$$
$$c = \sqrt{31}$$

$$c^2 = a^2 + 1^2 - 2a\cos(120°)$$
$$c^2 = a^2 + 1^2 + a$$
$$2c\,dc/dt = 2a\,da/dt + da/dt$$
$$2\sqrt{31}\,dc/dt = 2(5)(300) + 300$$

$$\boxed{dc/dt = \frac{3300}{2\sqrt{31}} = \frac{1650}{\sqrt{31}} \text{ Km/hr}}$$

(Q12)



Looking for $d\theta/dt$ when $\theta = \pi/4$

Given $dx/dt = 2$

$$\sin\theta = \frac{x}{10}$$

$$\cos\theta \, \frac{d\theta}{dt} = \frac{1}{10} \frac{dx}{dt}$$

$$\cos(\pi/4) \, d\theta/dt = \frac{1}{10}(2)$$

$$d\theta/dt = \frac{1}{5}\left(\frac{\sqrt{2}}{1}\right)$$

$$\boxed{d\theta/dt = \sqrt{2}/5 \text{ rad/s}}$$