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Test 1

This Test is graded out of 50. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* Simplify:

$$\frac{(-3xy^{-2}z^0)^{-3}}{(2x^2y^{-1}(xy)^{-1}z^2)^2} = \frac{(-3)^{-3}x^{-3}y^6}{2^2x^4y^{-2}(xy)^{-2}z^4}$$

$$= \frac{y^8}{-27\cdot 4\, x^7x^{-2}y^{-2}z^4}$$

$$= \frac{-y^{10}}{108\,x^5z^4}$$

Question 2. *(3 marks)* Expand and simplify:

$$x^2(3x-1)^2 = x^2[9x^2-6x+1]$$
$$= 9x^4 - 6x^3 + x^2$$

Question 3. *(3 marks)* Use long division to find the quotient and remainder:

$$\frac{x^3+2x^2-4}{x-3}$$

$$\begin{array}{r} x^2+5x+15 \\ x-3 \overline{)\, x^3+2x^2+0x-4} \\ -(x^3-3x^2) \\ \hline 5x^2+0x \\ -(5x^2-15x) \\ \hline 15x-4 \\ -(15x-45) \\ \hline 41 \end{array}$$

$$\therefore \frac{x^3+2x^2-4}{x-3} = x^2+5x+15 + \frac{41}{x-3}$$

Question 4. *(1 mark)* Factor:

$16-9x^2 = 4^2 - 3^2x^2$

$= 4^2 - (3x)^2$

$= (4-3x)(4+3x)$

Question 5. *(2 marks)* Factor:

$4x^2-12x+9 = (2x-3)(2x-3)$

$= (2x-3)^2$

Question 6. *(1 mark)* Factor:

$x^2-13x+42 = (x-6)(x-7)$

Question 7. *(2 mark)* Factor (hint: first by grouping):

$x^3-3x^2-4x+12 = x^2(x-3)-4(x-3)$

$= (x-3)(x^2-4)$

$= (x-3)(x-2)(x+2)$

Question 8. *(3 marks)* Factor:

$3x^3-24x^2+48x = 3x(x^2-8x+16)$

$= 3x(x-4)^2$

Question 9. *(5 marks)* Simplify:

$$\frac{x^2-1}{2x-4} \times \frac{x^2-4}{x^2-x-2} \times \frac{3x-6}{x^2+x-2}$$

$$= \frac{(x+1)(x-1)}{2(x-2)} \cdot \frac{(x-2)(x+2)}{(x-2)(x+1)} \cdot \frac{3(x-2)}{(x+2)(x-1)}$$

$$= \frac{3}{2}$$

Question 10. *(5 marks)* Simplify:

$$\frac{x}{x-2} + \frac{4+2x}{x^2-4} = \frac{x}{x-2} + \frac{4+2x}{(x-2)(x+2)} \qquad LCD = (x-2)(x+2)$$

$$= \frac{x(x+2)}{(x-2)(x+2)} + \frac{4+2x}{(x-2)(x+2)}$$

$$= \frac{x^2+2x+4+2x}{(x-2)(x+2)}$$

$$= \frac{x^2+4x+4}{(x-2)(x+2)} = \frac{(x+2)(x+2)}{(x-2)(x+2)}$$

$$= \frac{(x+2)}{(x-2)}$$

Question 11. *(3 marks)* Simplify:

$$\sqrt{20}+\sqrt{45}+\sqrt{80} = \sqrt{4\cdot 5} + \sqrt{9\cdot 5} + \sqrt{16\cdot 5}$$

$$= 2\sqrt{5} + 3\sqrt{5} + 4\sqrt{5}$$

$$= 9\sqrt{5}$$

Question 12. *(2 marks)* Solve for x:

$4(x-1)=x+17$

$4x-4 = x+17$

$3x = 21$

$x = 7$

Question 13. *(2 marks each)* Rationalize the denominator:

a.

$\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$

b.

$\frac{1}{1+\sqrt{2}}\left(\frac{1-\sqrt{2}}{1-\sqrt{2}}\right) = \frac{1-\sqrt{2}}{1-2} = \sqrt{2}-1$

Question 14. *(2 marks)* Solve for x by factoring:

$2x^2 = 8x$

$0 = 2x^2 - 8x$

$0 = 2x(x-4)$

$2x=0 \quad x-4=0$

$x=0 \quad x=4$

$\therefore x = 0, 4$

Question 15. *(3 marks)* Solve for x using the quadratic formula:

$x^2 = 10x+5$

$0 = x^2 - 10x - 5$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{10 \pm \sqrt{(-10)^2-4(1)(-5)}}{2}$

$= \frac{10 \pm \sqrt{100+20}}{2}$

$= \frac{10 \pm \sqrt{120}}{2}$

$= \frac{10 \pm 2\sqrt{30}}{2}$

$= 5 \pm \sqrt{30}$

Question 16. *(3 marks)* Find the quadratic equation such that 2 and 3 are its solution:

$$(x-x_1)(x-x_2)=0$$
$$(x-2)(x-3)=0$$
$$x^2-5x+6=0$$

Question 17. *(5 marks)* Solve for x:

$$\frac{5x^2}{x^2-4}+\frac{3}{2-x}=\frac{5x-1}{x+2}$$

LCM $=(x-2)(x+2)$

$$\frac{5x-1}{x+2}=\frac{5x^2}{(x-2)(x+2)}-\frac{3}{x-2}$$

$$\frac{(5x-1)(x-2)\cancel{(x+2)}}{\cancel{(x+2)}}=\frac{5x^2\cancel{(x-2)(x+2)}}{\cancel{(x-2)(x+2)}}-\frac{3\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$$

$$(5x-1)(x-2)=5x^2-3x-6$$

$$5x^2-10x-1+2=5x^2-3x-6$$

$$7=7x$$

$$1=x$$

Is the solution valid?

$x+2\neq 0$: $1+2\neq 0$

$2-x\neq 0$: $2-1\neq 0$

$x^2-4\neq 0$: $1^2-4\neq 0$

$\therefore x=1$

Bonus

Prove that $x^3 - 8 = 0$ has exactly one real solution. Follow the following steps:

a. *(1 mark)* Find r_1: the real solution of $x^3 - 8 = 0$

b. *(2 marks)* Using long division divide the factor $x - r_1$ from $x^3 - 8$.

c. *(1 marks)* Rewrite the equation $x^3 - 8 = 0$ in factored form using the divisor and quotient obtained above.

d. *(2 mark)* Show that $x^3 - 8 = 0$ only has one real solution using the discriminant.

a) $x^3 = 8$

$x = \sqrt[3]{8}$

$x = 2$

$\therefore r_1 = 2$

b)

$$x-2 \overline{)\, x^3 + 0x^2 + 0x - 8}$$

$x^2 + 2x + 4$

$-(x^3 - 2x^2)$

$2x^2 + 0x$

$-(2x^2 - 4x)$

$4x - 8$

$-(4x - 8)$

0

c) $0 = x^3 - 8$

$0 = (x-2)(x^2+2x+4)$

since $\dfrac{x^3-8}{x-2} = x^2 + 2x + 4$

d) $0 = (x-2)(x^2+2x+4)$

$x - 2 = 0$

$x = 2$

↑ a real solution

$x^2 + 2x + 4 = 0$

$\Delta = b^2 - 4ac$

$= (2)^2 - 4(1)(4) < 0$

$\therefore$ no real solutions

$\therefore x = 2$ the only real solution.