

## Test 1

This Test is graded out of 50. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.** (3 marks) Simplify:

$$\begin{aligned} \frac{(-2xy^2z^0)^{-2}}{(3x^3y^{-2}(xy)^{-2}z^3)^{-3}} &= \frac{(-2)^{-2}x^{-2}y^{-4}}{(3)^{-3}x^{-9}y^6(xy)^6z^{-9}} \\ &= \frac{27x^9z^9}{4x^2y^4y^6x^6y^6} \\ &= \frac{27xz^9}{4y^{16}} \end{aligned}$$

**Question 2.** (3 marks) Expand and simplify:

$$\begin{aligned} 2x^2(x-2)^2 &= 2x^2[x^2-4x+4] \\ &= 2x^4-8x^3+8x^2 \end{aligned}$$

**Question 3.** (3 marks) Use long division to find the quotient and remainder:

$$\frac{5x^3+x^2+4}{x+1}$$

$$\begin{array}{r} 5x^2 - 4x + 4 \\ x+1 \overline{) 5x^3 + x^2 + 0x + 4} \\ \underline{-(5x^3 + 5x^2)} \phantom{+ 4} \\ -4x^2 + 0x \phantom{+ 4} \\ \underline{-(-4x^2 - 4x)} \phantom{+ 4} \\ 4x + 4 \\ \underline{-(4x + 4)} \\ 0 \end{array}$$

$$\frac{5x^3+x^2+4}{x+1} = 5x^2-4x+4$$

**Question 4.** (1 mark) Factor:

$$\begin{aligned}25 - 9x^2 &= 5^2 - 3^2x^2 \\ &= 5^2 - (3x)^2 \\ &= (5 - 3x)(5 + 3x)\end{aligned}$$

**Question 5.** (2 marks) Factor:

$$\begin{aligned}4x^2 - 20x + 25 &= (2x + 5)(2x + 5) \\ &= (2x + 5)^2\end{aligned}$$

**Question 6.** (1 mark) Factor:

$$x^2 - 2x - 63 = (x - 9)(x + 7)$$

**Question 7.** (2 mark) Factor (hint: first by grouping):

$$\begin{aligned}12x^5 + 12x^3 - 4x^4 - 4x^2 &= 12x^3(x^2 + 1) - 4x^2(x^2 + 1) \\ &= (x^2 + 1)(12x^3 - 4x^2) \\ &= 4x^2(x^2 + 1)(3x - 1)\end{aligned}$$

**Question 8.** (3 marks) Factor:

$$\begin{aligned}15x^4 - 25x^3 + 10x^2 &= 5x^2(3x^2 - 5x + 2) \\ &= 5x^2(3x^2 - 2x - 3x + 2) \\ &= 5x^2(x(3x - 2) - 1(3x - 2)) \\ &= 5x^2((x - 1)(3x - 2))\end{aligned}$$

$$\begin{aligned}3x^2(2) &= 6x^2 = ab \\ \text{s.t. } a + b &= -5x \\ -2x - 3x &= -5x\end{aligned}$$

Question 9. (5 marks) Simplify:

$$\begin{aligned} & \frac{x^2-x-6}{2x-4} \times \frac{x^2-x-2}{x^2-4x} \times \frac{4x-16}{x^2+x} \\ & = \frac{(x-3)(x+2)}{2(x-2)} \cdot \frac{(x-2)(x+1)}{x(x-4)} \cdot \frac{4(x-4)}{x(x+1)} \\ & = \frac{2(x-3)(x+2)}{x^2} \end{aligned}$$

Question 10. (5 marks) Simplify:

$$\frac{x}{x-1} - \frac{2}{x^2-1} = \frac{x}{x-1} - \frac{2}{(x-1)(x+1)}$$

$$\text{LCD} = (x-1)(x+1)$$

$$\begin{aligned} & = \frac{x(x+1)}{(x-1)(x+1)} - \frac{2}{(x-1)(x+1)} \\ & = \frac{x^2+x-2}{(x-1)(x+1)} \\ & = \frac{(x+2)(x-1)}{(x-1)(x+1)} = \frac{(x+2)}{(x+1)} \end{aligned}$$

Question 11. (3 marks) Simplify:

$$\begin{aligned} 4\sqrt{12} - \sqrt{27} + 2\sqrt{48} & = 4\sqrt{4 \cdot 3} - \sqrt{9 \cdot 3} + 2\sqrt{16 \cdot 3} \\ & = 4(2\sqrt{3}) - 3\sqrt{3} + 2(4\sqrt{3}) \\ & = 8\sqrt{3} - 3\sqrt{3} + 8\sqrt{3} \\ & = 13\sqrt{3} \end{aligned}$$

Question 12. (2 marks) Solve for x:

$$5 - (8 + 3x) = 2x + 3$$

$$5 - 8 - 3x = 2x + 3$$

$$-6 = 5x$$

$$-\frac{6}{5} = x$$

Question 13. (2 marks each) Rationalize the denominator:

a.

$$\frac{1}{\sqrt{3}} \left( \frac{\sqrt{3}}{\sqrt{3}} \right) = \frac{\sqrt{3}}{3}$$

b.

$$\frac{1}{\sqrt{3}-1} \left( \frac{\sqrt{3}+1}{\sqrt{3}+1} \right) = \frac{\sqrt{3}+1}{3-1} = \frac{\sqrt{3}+1}{2}$$

Question 14. (2 marks) Solve for x by factoring:

$$2x^2 = -6x$$

$$0 = 2x^2 + 6x$$

$$0 = 2x(x+3)$$

$$\begin{array}{l} \swarrow \quad \searrow \\ 2x = 0 \quad x+3 = 0 \\ x = 0 \quad x = -3 \end{array}$$

Question 15. (3 marks) Solve for x using the quadratic formula:

$$x^2 = 100 - 21x$$

$$0 = x^2 + 21x - 100$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-21 \pm \sqrt{(21)^2 - 4(1)(-100)}}{2} \end{aligned}$$

$$= \frac{-21 \pm 29}{2} = -25 \text{ and } 4$$

$$\therefore x = -25 \text{ and } x = 4$$

Question 16. (3 marks) Find the quadratic equation such that 1 and 2 are its solutions:

$$\begin{aligned}(x-x_1)(x-x_2) &= 0 \\ 0 &= (x-1)(x-2) \\ 0 &= x^2 - 3x + 2\end{aligned}$$

Question 17. (5 marks) Solve for x:

$$\frac{3}{3+x} - \frac{x}{3-x} = \frac{x^2+9}{x^2-9}$$

$$\frac{x^2+9}{(x-3)(x+3)} = \frac{3}{3+x} + \frac{x}{x-3}$$

$$\text{LCD} = (x-3)(x+3)$$

$$\frac{(x^2+9)\cancel{(x-3)}\cancel{(x+3)}}{\cancel{(x-3)}\cancel{(x+3)}} = \frac{3\cancel{(x+3)}(x-3)}{\cancel{(3+x)}} + \frac{x\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$$

$$x^2+9 = 3x-9 + x^2+3x$$

$$18 = 6x$$

$$3 = x$$

Verify solution

$$3-x: 3-3=0$$

not a valid solution hence no solutions.

**Bonus**

Prove that  $x^3 - 8 = 0$  has exactly one real solution. Follow the following steps:

- (1 mark) Find  $r_1$ : the real solution of  $x^3 - 8 = 0$
- (2 marks) Using long division divide the factor  $x - r_1$  from  $x^3 - 8$ .
- (1 marks) Rewrite the equation  $x^3 - 8 = 0$  in factored form using the divisor and quotient obtained above.
- (2 mark) Show that  $x^3 - 8 = 0$  only has one real solution using the discriminant.

$$\begin{aligned} \text{a)} \quad x^3 &= 8 \\ x &= \sqrt[3]{8} \\ x &= 2 \\ \therefore r_1 &= 2 \end{aligned}$$

$$\begin{array}{r} \text{b)} \quad x-2 \overline{) \begin{array}{r} x^2 + 2x + 4 \\ x^3 + 0x^2 + 0x - 8 \\ \underline{-(x^3 - 2x^2)} \\ 2x^2 + 0x \\ \underline{-(2x^2 - 4x)} \\ 4x - 8 \\ \underline{-(4x - 8)} \\ 0 \end{array}} \end{array}$$

$$\begin{aligned} \text{c)} \quad 0 &= x^3 - 8 \\ 0 &= (x-2)(x^2 + 2x + 4) \end{aligned}$$

$$\text{Since } \frac{x^3 - 8}{x-2} = x^2 + 2x + 4$$

$$\text{d)} \quad 0 = (x-2)(x^2 + 2x + 4)$$

$$\begin{array}{l} \downarrow \\ x-2=0 \\ x=2 \end{array}$$

$\uparrow$   
a real solution

$$\begin{array}{l} \downarrow \\ x^2 + 2x + 4 = 0 \end{array} \quad \Delta = b^2 - 4ac = (2)^2 - 4(1)(4) < 0$$

$\therefore$  no real solutions

$\therefore x=2$  the only real solution.