Dawson College: **Functions and Trigonometry**: 201-009-50-C02: Fall 2008

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Test 2

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Let $f(x) = -2x^2 + 3x + 2$, then find

a. *(1 marks)* $f(4)$

b. *(3 marks)* x if $f(x) = 2$

a) $f(4) = -2(4)^2 + 3(4) + 2 = -18$

b) $f(x) = 2$

$-2x^2 - 3x + 2 = 2$

$0 = -2x^2 + 3x$

$0 = x(-2x + 3)$

$x = 0$

$-2x + 3 = 0$

$2x = +3$

$x = +\frac{3}{2}$

Question 2. *(1 mark)* If $f(x)$ is injective and $f(5) = 0$ then find $f^{-1}(0)$.

$f^{-1}(0) = 5$

Question 3. *(4 marks)* Find the equation of the line that passes through the point $(1,2)$ and $(5,6)$.

$y = mx + b$

$m = \frac{\Delta y}{\Delta x}$

$= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{6-2}{5-1}$

$= \frac{4}{4}$

$= 1$

$y = x + b$

$2 = 1 + b$

$1 = b$

$\therefore y = x + 1$

Question 4.

a. *(4 marks)* Find the *distance* and the *midpoint* of the line segment joining the points $(2,1)$ and $(5,3)$.

b. *(2 marks)* Find the equation of the circle whose center is $(2,3)$ and has a radius of 5.

a) $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(5-2)^2+(3-1)^2}$

$=\sqrt{3^2+2^2}$

$=\sqrt{9+4}$

$=\sqrt{13}$

$(x_m, y_m)=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$=\left(\frac{2+5}{2}, \frac{1+3}{2}\right)$

$=\left(\frac{7}{2}, 2\right)$

b) $(x-h)^2+(y-k)^2=5^2$

$(x-2)^2+(y-3)^2=25$

Question 5. *(4 marks)* Use the x and y intercepts to sketch the graph the linear function.

$3x-2y=18$

x-int:

$y=0$

$3x=18$

$x=6$

$\therefore (6,0)$

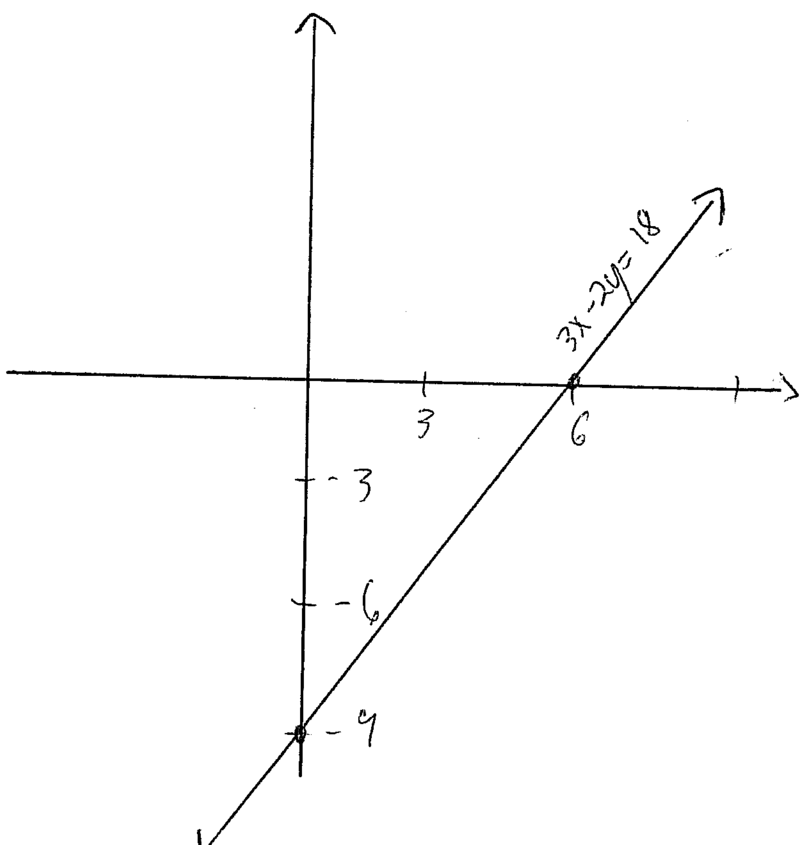
y-int:

$x=0$

$-2y=18$

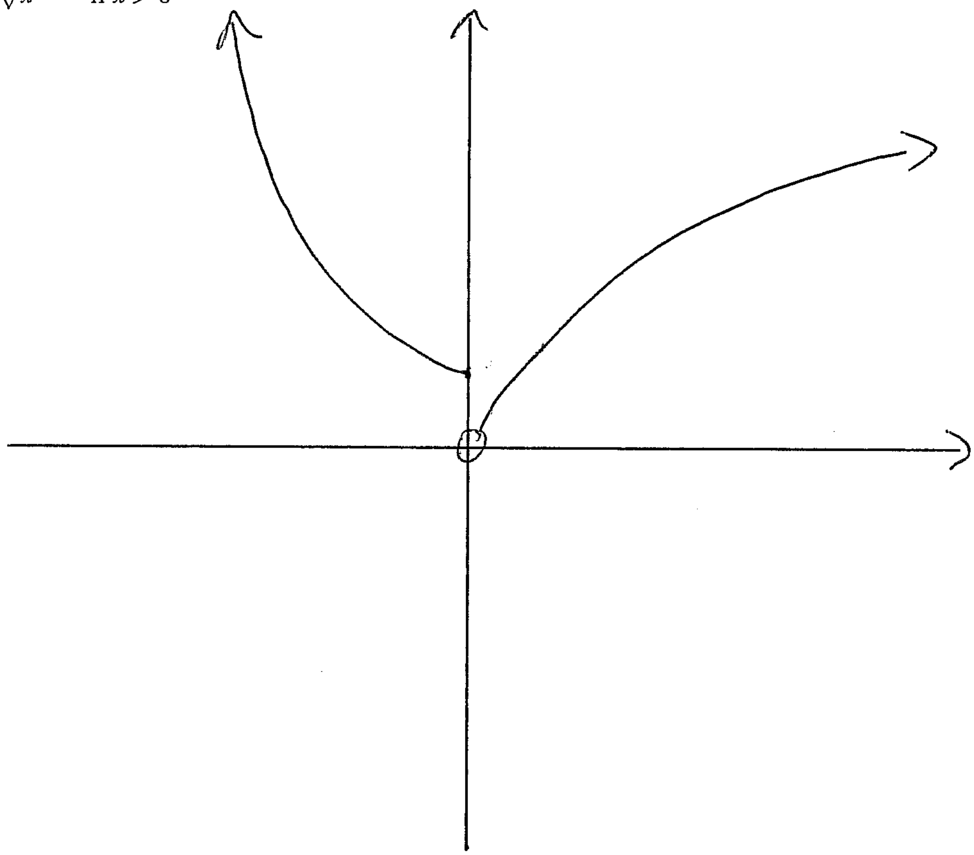
$y=-9$

$\therefore (0,-9)$



Question 8. *(4 marks)* Sketch the graph defined by

$$f(x) = \begin{cases} x^2+1 & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$$



Question 9. *(4 marks)* Find the equation of the line that passes through the point $(2,3)$ and is parallel to the line $x+2y=4$.

$2y = -x + 4$

$y = \frac{-x}{2} + 2$

$m = \frac{-1}{2}$

$\therefore \quad y = \frac{-1}{2}x + b$

$3 = \frac{-1}{2}(2) + b$

$3 = -1 + b$

$4 = b$

$\therefore \quad y = \frac{-1}{2}x + 4$

Question 6. Let $f(x) = 2x^2 - 2x + 1$ and $g(x) = \frac{x}{4x+1}$.

a. *(4 marks)* Determine $\frac{f(x+h)-f(x)}{h}$ and simplify.

b. *(1 marks)* Determine the domain of $g(x)$.

c. *(2 marks)* Determine $(f \circ g)(x)$ and $(g \circ f)(x)$. *Do not simplify.*

d. *(2 marks)* Determine $(g \circ f)(1)$.

e. *(bonus 1 mark)* Determine the range of $g(x)$.

a) $$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2 - 2(x+h) + 1 - [2x^2 - 2x + 1]}{h}$$

$$= \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{2x} - 2h + \cancel{1} - \cancel{2x^2} + \cancel{2x} - \cancel{1}}{h}$$

$$= \frac{4xh + 2h^2 - 2h}{h} = \frac{\cancel{h}(4x + 2h - 2)}{\cancel{h}}$$

$$= 4x + 2h - 2$$

b) $4x + 1 \neq 0$

$4x \neq -1$

$x = \frac{-1}{4}$

$\therefore$ all reals except $\frac{-1}{4}$

c) $(f \circ g)(x) = f(g(x))$

$= f\left(\frac{x}{4x+1}\right)$

$= 2\left(\frac{x}{4x+1}\right)^2 - 2\left(\frac{x}{4x+1}\right) + 1$

$(g \circ f)(x) = g(f(x))$

$= g(2x^2 - 2x + 1)$

$= \frac{2x^2 - 2x + 1}{4(2x^2 - 2x + 1) + 1}$

d) $(g \circ f)(1) = g(f(1))$

$= g(2(1)^2 - 2(1) + 1)$

$= g(2 - 2 + 1)$

$= g(1)$

$= \frac{1}{4+1} = \frac{1}{5}$

e) range of $g(x)$ is all the real numbers except $\frac{1}{4}$

Question 7. Let $f(x) = x^2 + 2x - 5$ be a quadratic function.

a. *(2 marks)* Determine the vertex of $f(x)$.

b. *(1 mark)* Determine the orientation of the parabola and state whether the vertex is a minimum or maximum.

c. *(1 mark)* Determine the y-intercept.

d. *(2 marks)* Determine the x-intercept(s).

e. *(1 mark)* Sketch the graph of $f(x)$.

f. *(1 mark)* Determine if $f(x)$ is injective and justify.

g. *(2 marks)* Determine the domain and range of $f(x)$.

b) $a>0$ ∪ ∴ vertex a min.

c) $(0, f(0)) = (0, c) = (0, -5)$

a) $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) = \left(\frac{-2}{2(1)}, f\left(\frac{-2}{2}\right)\right) = (-1, f(-1)) = (-1, -6)$

d) $0 = f(x)$

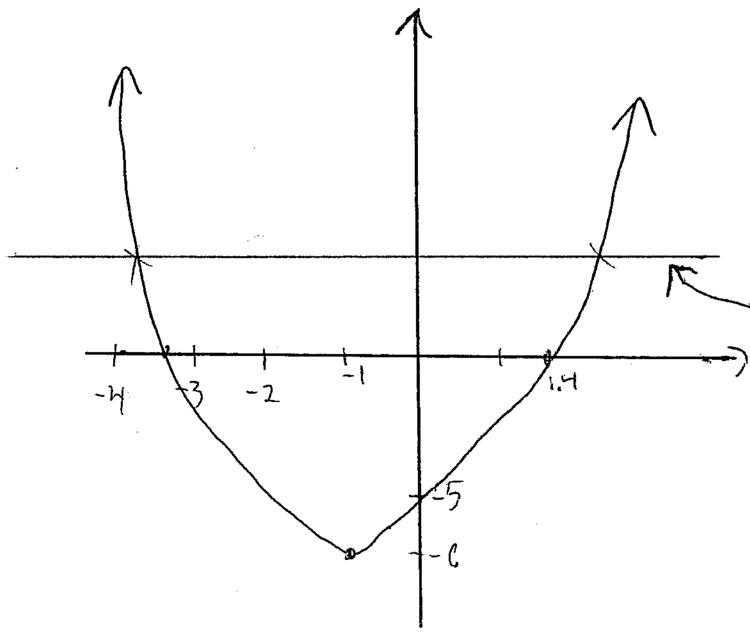
$0 = x^2 + 2x - 5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-5)}}{2}$

$= \frac{-2 \pm \sqrt{24}}{2}$

$\doteq 1.4$ and -3.4

e)



f) not injective since it fails the horizontal line test.

g) Domain: $\mathbb{R}$

Range: $[-6, \infty)$

Question 10. *(4 marks)* If $f(x) = \frac{x+1}{x-2}$ then find $f^{-1}(x)$.

$$y = \frac{x+1}{x-2}$$

$$x = \frac{y+1}{y-2}$$

$$x(y-2) = y+1$$

$$xy - 2x = y+1$$

$$xy - y = 2x+1$$

$$y(x-1) = 2x+1$$

$$y = \frac{2x+1}{x-1}$$

$$\therefore f^{-1}(x) = \frac{2x+1}{x-1}$$

Bonus. *(4 marks)* Find the quadratic function whose graph passes through the points $(0,4)$, $(1,4)$ and $(-1,6)$.

$$f(x) = ax^2 + bx + c$$

$$4 = f(0)$$
$$4 = a(0)^2 + b(0) + c$$
$$4 = c$$

$$4 = f(1)$$
$$4 = a(1)^2 + b(1) + 4$$
$$4 = a + b + 4$$
$$a = -b \quad (*)$$

$$6 = f(-1)$$
$$6 = a(-1)^2 + b(-1) + 4$$
$$2 = +a - b \quad \text{sub } (*)$$
$$2 = -b - b$$
$$2 = -2b$$
$$-1 = b$$

$$\therefore a = 1$$

$$\therefore f(x) = x^2 - x + 4$$