Dawson College: **Functions and Trigonometry**: 201-009-50-C02: Fall 2008

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Test 2

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.

a. *(4 marks)* Find the *distance* and the *midpoint* of the line segment joining the points $(1,2)$ and $(6,3)$.

b. *(2 marks)* Find the equation of the circle whose center is $(1,2)$ and has a radius of 5.

a)
$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(6-1)^2+(3-2)^2}$$
$$=\sqrt{25+1}$$
$$=\sqrt{26}$$

$$(x_m, y_m)=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{1+6}{2}, \frac{2+3}{2}\right)=\left(\frac{7}{2}, \frac{5}{2}\right)$$

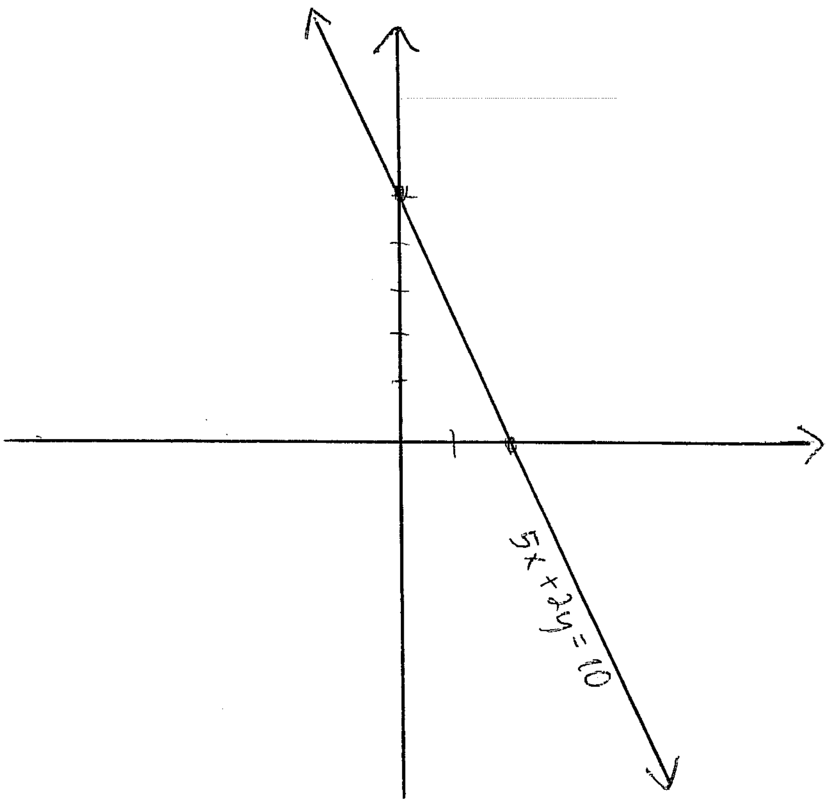
b)
$$(x-h)^2+(y-k)^2=5^2$$
$$(x-1)^2+(y-2)^2=25$$

Question 2. *(4 marks)* Use the x and y intercepts to sketch the graph the linear function.

$5x+2y=10$

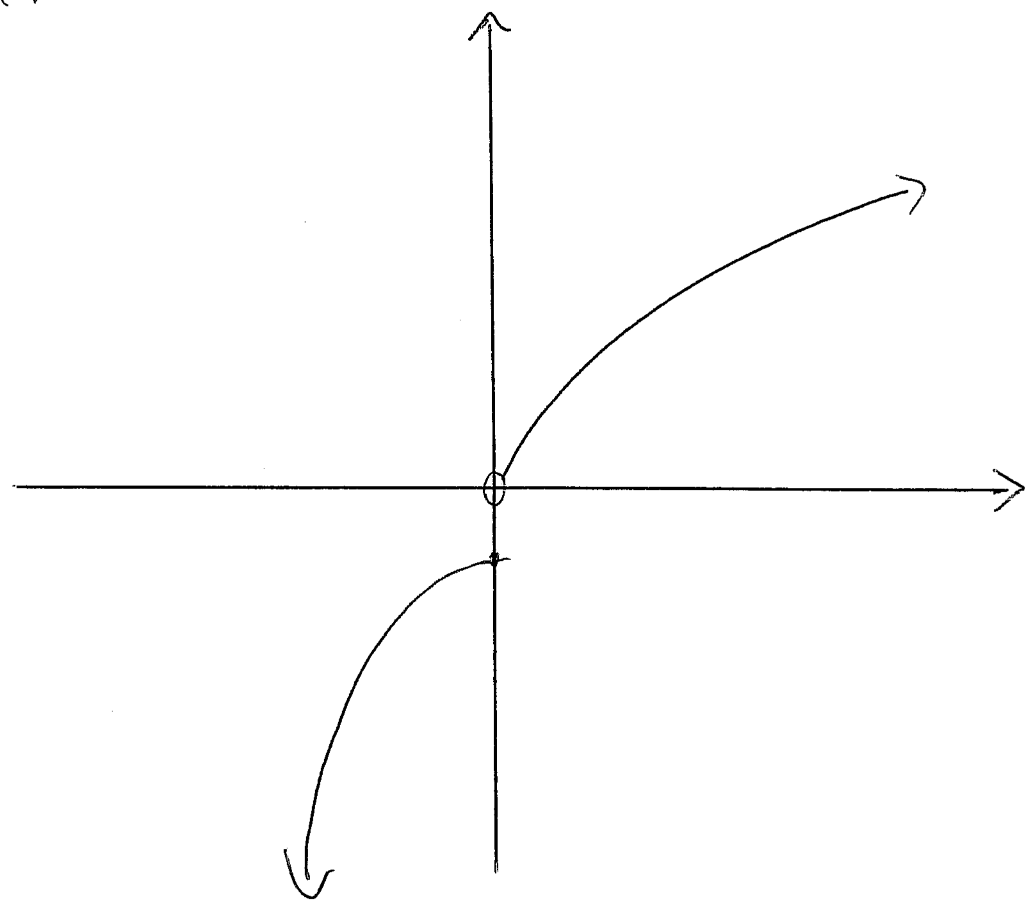
x-int:
let $y=0$
$5x=10$
$x=2$
$\therefore (2,0)$

y-int:
let $x=0$
$2y=10$
$y=5$
$\therefore (0,5)$



Question 3. *(4 marks)* Sketch the graph defined by

$$f(x) = \begin{cases} -x^2 - 1 & \text{if } x \le 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$$



Question 4. *(4 marks)* Find the equation of the line that passes through the point $(3, 1)$ and is parallel to the line $x + 3y = 6$.

$3y = -x + 6$

$y = \frac{-x}{3} + 2$

$\therefore \quad m = \frac{-1}{3}$

$\therefore \quad y = -\frac{1}{3}x + 2$

$y = mx + b$

$1 = -\frac{1}{3}(3) + b$

$1 = -1 + b$

$2 = b$

Question 5. Let $f(x) = -2x^2 + 3x + 1$ and $g(x) = \frac{x}{2x+2}$.

a. *(4 marks)* Determine $\frac{f(x+h)-f(x)}{h}$ and simplify.

b. *(1 marks)* Determine the domain of $g(x)$.

c. *(2 marks)* Determine $(f \circ g)(x)$ and $(g \circ f)(x)$. *Do not simplify.*

d. *(2 marks)* Determine $(g \circ f)(1)$.

e. *(bonus 1 mark)* Determine the range of $g(x)$.

b) $2x + 2 \neq 0$
$x \neq -1$
$\therefore$ all real numbers except $x = -1$

a) $$\frac{f(x+h)-f(x)}{h} = \frac{-2(x+h)^2 + 3(x+h) + 1 - [-2x^2 + 3x + 1]}{h}$$

$$= \frac{\cancel{-2x^2} - 4xh - 2h^2 \cancel{+3x} + 3h \cancel{+1} \cancel{+2x^2} \cancel{-3x} \cancel{-1}}{h}$$

$$= \frac{-4xh - 2h^2 + 3h}{h} = \frac{\cancel{h}(-4x - 2h + 3)}{\cancel{h}}$$

$$= -4x - 2h + 3$$

c) $$(f \circ g)(x) = f(g(x))$$
$$= f\left(\frac{x}{2x+2}\right)$$
$$= -2\left(\frac{x}{2x+2}\right)^2 + 3\left(\frac{x}{2x+2}\right) + 1$$

$$(g \circ f)(x) = g(f(x)) = g(-2x^2 + 3x + 1) = \frac{-2x^2 + 3x + 1}{2(-2x^2 + 3x + 1) + 2}$$

d) $$(g \circ f)(1) = g(f(1))$$
$$= g(-2(1)^2 + 3(1) + 1)$$
$$= g(2) = \frac{2}{2(2)+2} = \frac{2}{6} = \frac{1}{3}$$

e) The range is all real numbers except $\frac{1}{2}$

Question 6. Let $f(x) = -3x^2 + 5x + 4$, then find

a. *(1 marks)* $f(2)$

b. *(3 marks)* x if $f(x) = 4$

$$f(2) = -3(2)^2 + 5(2) + 4$$
$$= -3 \cdot 4 + 10 + 4$$
$$= 2$$

b)

$$4 = f(x)$$
$$4 = -3x^2 + 5x + 4$$
$$0 = 3x^2 - 5x$$
$$0 = x(3x - 5)$$

$x = 0$

$3x - 5 = 0$
$3x = 5$
$x = 5/3$

Question 7. *(1 mark)* If $f(x)$ is injective and $f(3) = 2$ then find $f^{-1}(2)$.

$$f^{-1}(2) = 3$$

Question 8. *(4 marks)* Find the equation of the line that passes through the point $(2,3)$ and $(4,7)$.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{4-2} = \frac{4}{2} = 2$$

$$y = 2x + b$$
$$3 = 2(2) + b$$
$$-1 = b$$

$$\therefore \; y = 2x - 1$$

Question 9. Let $f(x) = x^2 - 6x + 2$ be a quadratic function.

a. *(2 marks)* Determine the vertex of $f(x)$.

b. *(1 mark)* Determine the orientation of the parabola and state whether the vertex is a minimum or maximum.

c. *(1 mark)* Determine the y-intercept.

d. *(2 marks)* Determine the x-intercept(s).

e. *(1 mark)* Sketch the graph of $f(x)$.

f. *(1 mark)* Determine if $f(x)$ is injective and justify.

g. *(2 marks)* Determine the domain and range of $f(x)$.

b) $\cup$ since $a = 1 > 1$ and the vertex is a min

c) $(0, f(0)) = (0, c) = (0, 2)$

a) $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) = \left(\frac{-(-6)}{2(1)}, f\left(\frac{-(-6)}{2(1)}\right)\right) = (3, f(3)) = (3, 3^2 - 6(3) + 2)$

$= (3, 9 - 18 + 2)$

$= (3, -7)$

d) $0 = f(x)$

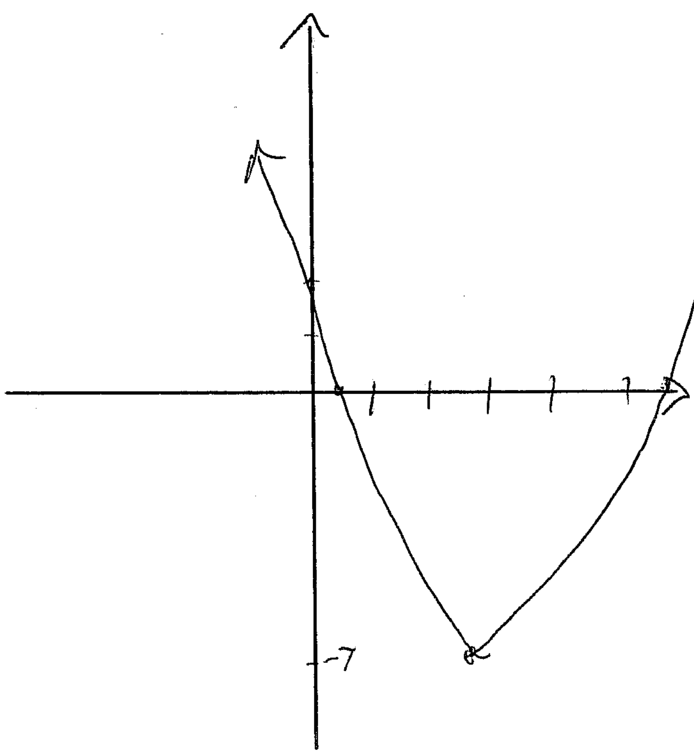
$0 = x^2 - 6x + 2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{6 \pm \sqrt{(6)^2 - 4(1)(2)}}{2} = \frac{6 \pm \sqrt{28}}{2}$

$\doteq 5.6$ and 0.3

e)



f) $f(x)$ is not injective since it fails the horizontal line test

g) Domain $\mathbb{R}$

Range: $[-7, \infty)$

Question 10. *(4 marks)* If $f(x) = \frac{x-3}{x-5}$ then find $f^{-1}(x)$.

$$y = \frac{x-3}{x-5}$$

$$x = \frac{y-3}{y-5}$$

$$x(y-5) = y-3$$

$$xy-5x = y-3$$

$$xy-y = 5x-3$$

$$y(x-1) = 5x-3$$

$$y = \frac{5x-3}{x-1}$$

$$\therefore \quad f^{-1}(x) = \frac{5x-3}{x-1}$$

Bonus. *(4 marks)* Find the quadratic function whose graph passes through the points $(0,5)$, $(1,5)$ and $(-1,7)$.

$$f(x) = ax^2 + bx + c$$

$$f(0) = 5$$

$$5 = a(0)^2 + b(0) + c$$

$$5 = c$$

$$f(1) = 5$$

$$5 = a(1)^2 + b(1) + 5$$

$$a = -b$$

$$f(-1) = 7$$

$$7 = a(-1)^2 + b(-1) + 5$$

$$7 = -b - b + 5$$

$$2 = -2b$$

$$-1 = b$$

$$\therefore \quad a = 1$$

$$\therefore \quad f(x) = x^2 - x + 5$$