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Test 3

This test is graded out of 48 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.

a. *(2 marks)* What angle θ ($0° \leq \theta < 360°$) is co-terminal to $1550°$.

b. *(2 marks)* Consider an angle θ in standard position. Then find the quadrant that its terminal edge lies in, if $\csc\theta < 0$ and $\cot\theta < 0$.

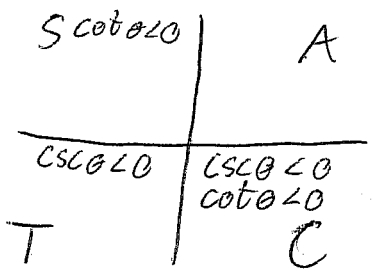
c. *(4 marks)* Find the values of the other trigonometric functions, if $\cos\theta = \frac{-1}{2}$ and $\tan\theta < 0$.

a) $\theta_2 = \theta_1 + k360$ $\quad k = \left\lfloor \frac{1550}{360} \right\rfloor = 4$

$1550 = \theta_1 + 4 \cdot 360$

$\theta_1 = 110°$

b)



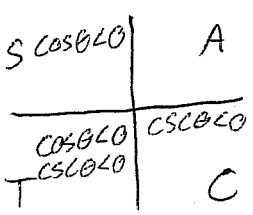
$\therefore$ terminal edge in 4th quadrant

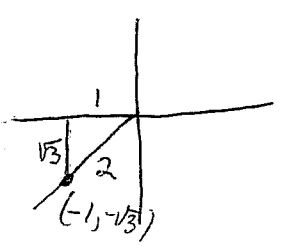
c) $\cos\theta = \frac{-1}{2} < 0$ and $\csc\theta < 0$

$\cos\theta = \frac{-1}{2} = \frac{adj.}{hyp.}$

$\Downarrow$

$\sec\theta = -2$





$\sin\theta = \frac{opp}{hyp} = -\frac{\sqrt{3}}{2} \Rightarrow \csc\theta = \frac{-2}{\sqrt{3}}$

$\tan\theta = \frac{opp.}{adj.} = \frac{\sqrt{3}}{1} \Rightarrow \cot\theta = \frac{1}{\sqrt{3}}$

Question 2.

a. *(4 marks)* Sketch the graph of $f(x) = 2^x + 1$.

b. *(4 marks)* Sketch the graph of $g(x) = \log_{\frac{1}{2}}(x+1)$

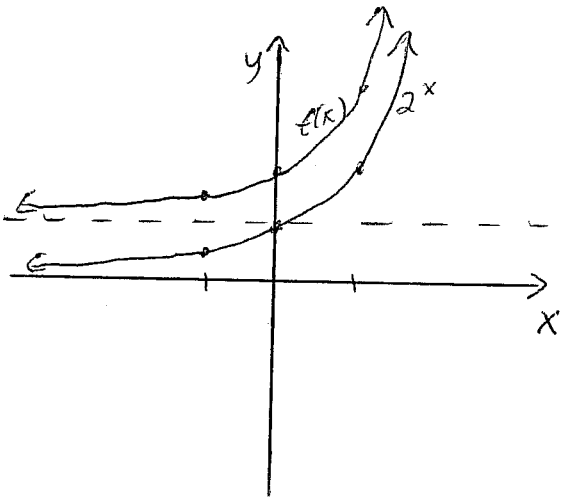
c. *(2 bonus marks)* State the domain and range of $f(x)$ and $g(x)$.

d. *(1 bonus mark)* Is $f(x)$ injective, justify.

a)

x	2^x
-1	$\frac{1}{2}$
0	1
1	2

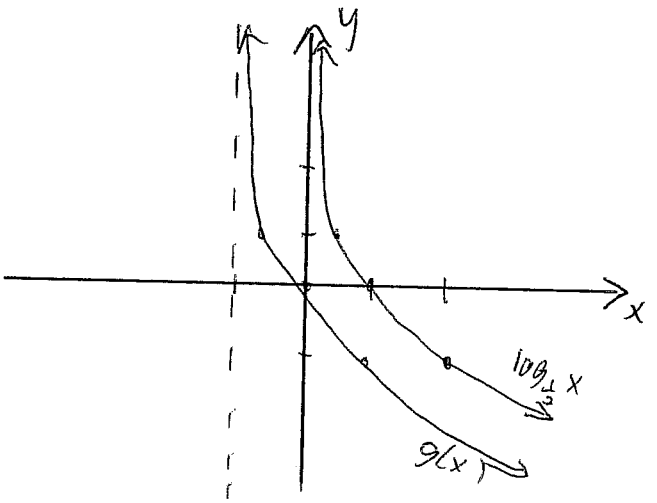
Domain: $\mathbb{R}$
Range: $(1, \infty)$



b)

x	$\log_{\frac{1}{2}} x$
2	-1
1	0
$\frac{1}{2}$	1

Domain: $(-1, \infty)$
Range: $(-\infty, \infty)$



d) yes, since any horizontal line intersect at most once.

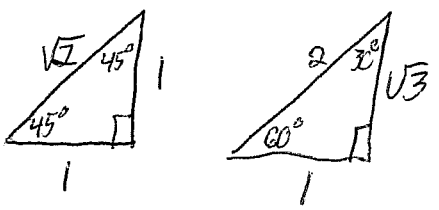
Question 3.

a. *(4 marks)* Draw the two "special triangle" which help identify the special angles. Label the angles of the triangles and the lengths of the sides.

b. *(4 marks)* Find the exact value of $\sec 945°$

c. *(4 marks)* Find the exact value of $\cos \frac{4\pi}{3}$

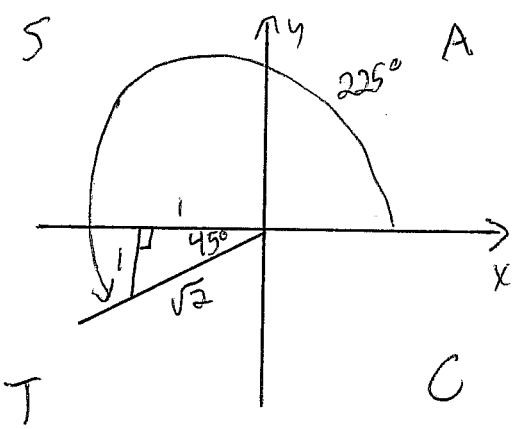
a)



b) $\sec 945° = \sec 225° = -\frac{hyp}{adj}$

$\therefore \theta_R = 45°$

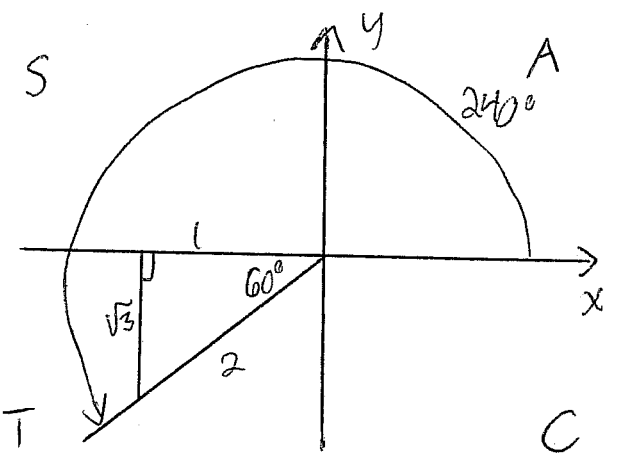
$= -\frac{\sqrt{2}}{1}$



c) $\cos \frac{4\pi}{3} = \cos \frac{4\pi}{3}\left(\frac{180}{\pi}\right) = \cos 240°$

$= -\frac{adj}{hyp}$

$= -\frac{1}{2}$



Question 4. Solve for x.

a. *(4 marks)*

$2^{2x-1} = 3^{3-x}$

b. *(4 marks)*

$log_2(x+1) + \log_2(x+4) = 2$

a) $\log_2 2^{2x-1} = \log_2 3^{3-x}$

$2x-1 = (3-x)\log_2 3$

$2x-1 = 3\log_2 3 - x\log_2 3$

$2x + x\log_2 3 = 3\log_2 3 + 1$

$x(2+\log_2 3) = 3\log_2 3 + 1$

$x = \dfrac{3\log_2 3 + 1}{(2+\log_2 3)}$

b) $\log_2 (x+1)(x+4) = 2$

$2^{\log_2 (x+1)(x+4)} = 2^2$

$(x+1)(x+4) = 4$

$x^2 + 5x + 4 = 4$

$x^2 + 5x = 0$

$x(x+5) = 0$

$x = 0 \qquad x = -5$ (not a solution)

$\therefore x = 0$

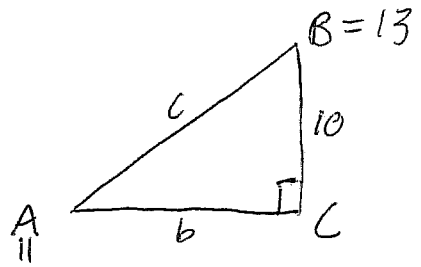
Question 5.

a. *(4 marks)* Solve the right triangle ABC ($C = 90°$) given: $a = 10$, $B = 13°$.

b. *(4 marks)* Solve for θ, giving the exact solution, $0° \leq \theta < 360°$

$\sqrt{3}\sec\theta + 2 = 0$

a)



$180 - 90 - 13 = 77°$

$\tan 77° = \frac{10}{b}$

$b = \frac{10}{\tan 77} \doteq 2.3$

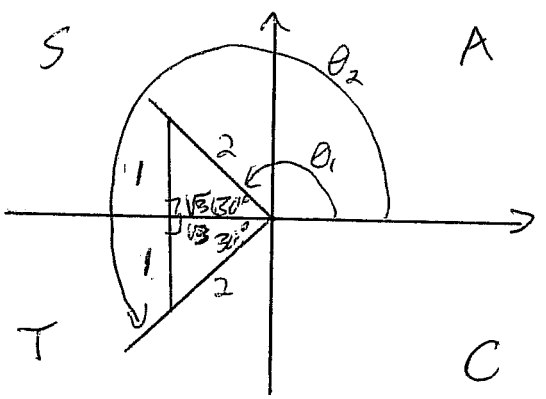
$c = \sqrt{10^2 + (2.3)^2}$

$\doteq 10.3$

b) $\sqrt{3}\sec\theta + 2 = 0$

$\sqrt{3}\sec\theta = -2$

$\sec\theta = \frac{-2}{\sqrt{3}} = \frac{hyp}{adj}$



$\theta_1 = 180° - 30° = 150°$

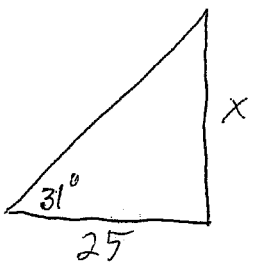
$\theta_2 = 180° + 30° = 210°$

Question 6. *(4 marks)* A tree casts a 25*m* long shadow when the angle of elevation of the sun is 31°. How tall is the tree?

$$\tan 31 = \frac{x}{25}$$

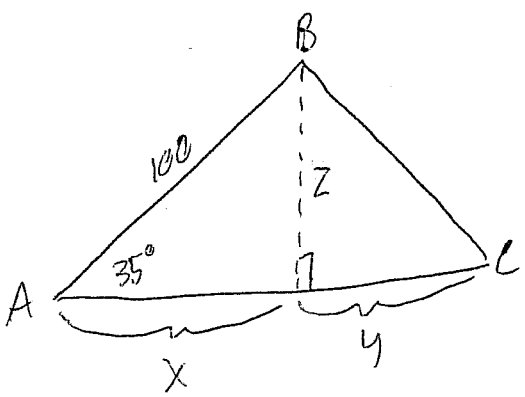
$$25 \tan 31 = x$$

$$x \doteq 15\,m$$



$\therefore$ the tree is 15m tall.

Bonus. *(3 marks)* Solve the triangle ABC where $A = 35°$, $a = 70$, $c = 100$. *Do not assume that an angle of the triangle is* 90°.



$$\sin 35 = \frac{Z}{100}$$

$$Z \doteq 57.4$$

$$\cos 35 = \frac{x}{100}$$

$$x \doteq 81.9$$

$$\sin C = \frac{Z}{70}$$

$$C = \sin^{-1}\left(\frac{57.4}{70}\right) \doteq 55°$$

$$70^2 = Z^2 + y^2$$

$$y = \sqrt{70^2 - (57.4)^2}$$

$$= 40$$

$$B = 180° - 35° - 55° = 90°$$

$\therefore b = 81.9 + 40 = 122$