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Test 3

This test is graded out of 48 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

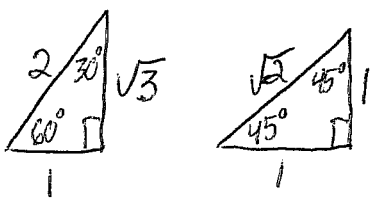
Question 1.

a. *(4 marks)* Draw the two "special triangle" which help identify the special angles. Label the angles of the triangles and the lengths of the sides.

b. *(4 marks)* Find the exact value of $\csc 675°$

c. *(4 marks)* Find the exact value of $\sin \frac{7\pi}{6}$

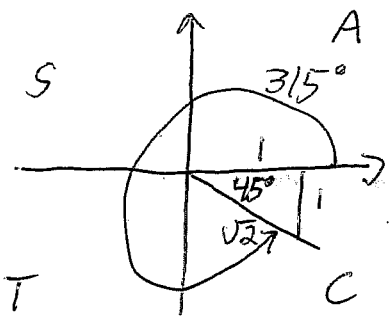
a)



b) $\csc 675° = \csc 315°$ $\quad \theta_R = 45°$

$= -\frac{hyp}{opp}$

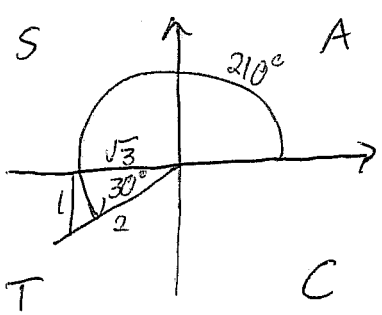
$= -\sqrt{2}$



c) $\sin \frac{7\pi}{6} = \sin \frac{7\pi}{6}\left(\frac{180}{\pi}\right) = \sin 210°$ $\quad \theta_R = 30°$

$= -\frac{opp}{hyp}$

$= -\frac{1}{2}$



Question 2.

a. *(2 marks)* What angle θ ($0° \leq \theta < 360°$) is co-terminal to $3070°$.

b. *(2 marks)* Consider an angle θ in standard position. Then find the quadrant that its terminal edge lies in, if $\sec\theta < 0$ and $\cot\theta < 0$.

c. *(4 marks)* Find the values of the other trigonometric functions, if $\sin\theta = \frac{-1}{2}$ and $\tan\theta < 0$.

a) $\theta_2 = \theta_1 + K360$ $\quad K = \left\lfloor \frac{3070}{360} \right\rfloor = 8$

$3070 = \theta_1 + 8 \cdot 360$

$\theta_1 = 190$

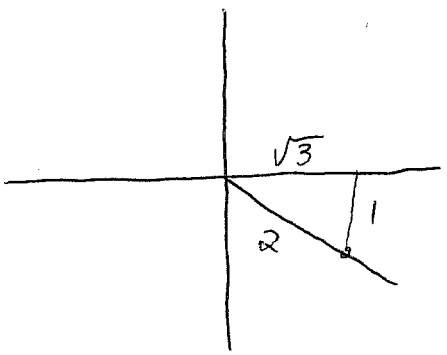
b)

S: $\sec\theta<0$, $\csc\theta<0$	A
$\sec\theta<0$	$\cot\theta<0$
T	C

$\therefore$ second quadrant

c) $\sin\theta = -\frac{1}{2} < 0$ and $\tan\theta < 0$

S: $\tan\theta<0$	A
$\sin\theta<0$	$\tan\theta<0$, $\sin\theta<0$
T	C



$\sin\theta = \frac{-1}{2} = \frac{opp}{hyp} \rightarrow \csc\theta = -2$

$\cos\theta = \frac{adj}{hyp} = \frac{\sqrt{3}}{2} \rightarrow \sec\theta = \frac{2}{\sqrt{3}}$

$\tan\theta = \frac{opp}{adj} = \frac{-1}{\sqrt{3}} \rightarrow \cot\theta = -\sqrt{3}$

Question 3.

a. *(4 marks)* Sketch the graph of $f(x) = \left(\frac{1}{2}\right)^x + 1$.

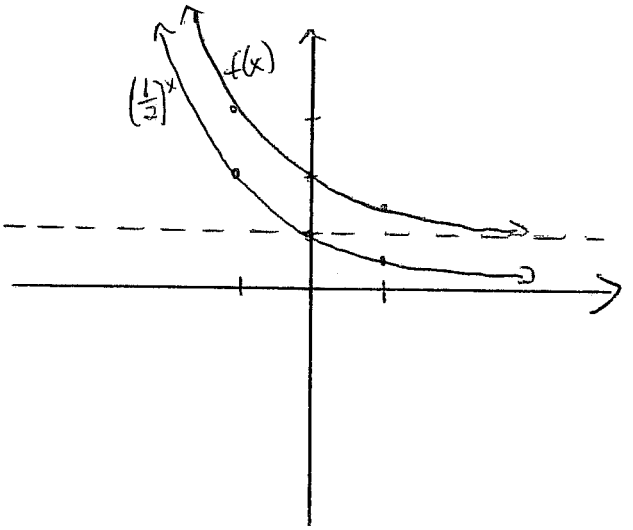
b. *(4 marks)* Sketch the graph of $g(x) = \log_2(x+1)$

c. *(2 bonus marks)* State the domain and range of $f(x)$ and $g(x)$.

d. *(1 bonus mark)* Is $f(x)$ injective, justify.

a)

x	$\left(\frac{1}{2}\right)^x$
-1	2
0	1
1	$\frac{1}{2}$

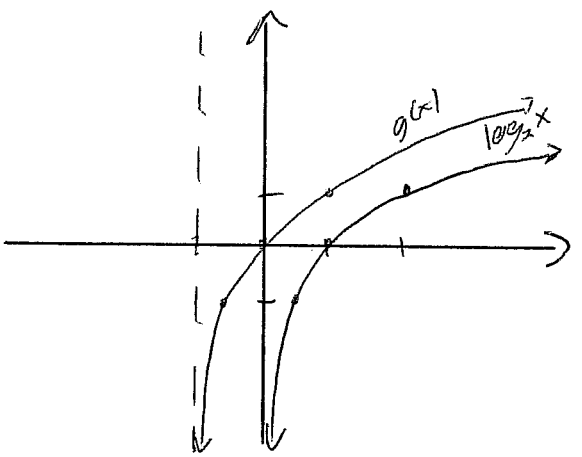


Domain : $\mathbb{R}$

Range : $(1, \infty)$

b)

x	$\log_2 x$
$\frac{1}{2}$	$\log_2\left(\frac{1}{2}\right) = -1$
1	$\log_2 1 = 0$
2	$\log_2 2 = 1$



Domain : $(1, \infty)$

Range : $\mathbb{R}$

d) $f(x)$ is injective since any horizontal line intersect at most once.

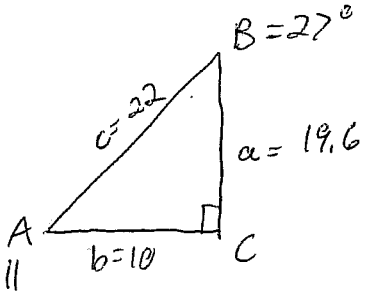
Question 4.

a. *(4 marks)* Solve the right triangle ABC ($C = 90°$) given: $b = 10$, $B = 27°$.

b. *(4 marks)* Solve for θ, giving the exact solution, $0° \leq \theta < 360°$

$\sqrt{3}\csc\theta + 2 = 0$

a)



$180 - 90 - 27$
$= 63°$

$\tan 63° = \frac{opp}{adj} = \frac{a}{10}$

$10\tan 63 = a$

$19.6 \doteq a$

$c^2 = a^2 + b^2$

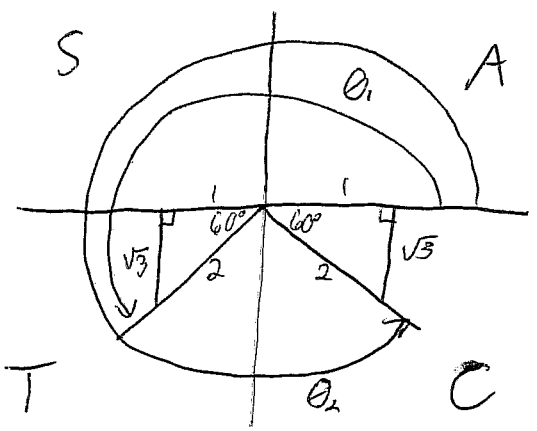
$c = \sqrt{10^2 + (19.6)^2}$

$c \doteq 22$

b)

$\sqrt{3}\csc\theta = -2$

$\csc\theta = \frac{-2}{\sqrt{3}} = \frac{hyp.}{opp.}$



$\theta_1 = 180° + 60°$

$= 240°$

$\theta_2 = 360° - 60°$

$= 300°$

Question 5. Solve for x.

a. *(4 marks)*

$3^{x-1} = 2^{3-2x}$

b. *(4 marks)*

$\log_2(x-3) + \log_2(x+3) = 4$

a) $3^{x-1} = 2^{3-2x}$

$\log_2 3^{x-1} = \log_2 2^{3-2x}$

$(x-1)\log_2 3 = 3-2x$

$x\log_2 3 - \log_2 3 = 3-2x$

$x\log_2 3 + 2x = 3 + \log_2 3$

$x(\log_2 3 + 2) = 3 + \log_2 3$

$x = \dfrac{3+\log_2 3}{\log_2 3 + 2}$

b)

$\log_2 (x-3)(x+3) = 4$

$\log_2 x^2 - 9 = 4$

$2^{\log_2 x^2 - 9} = 2^4$

$x^2 - 9 = 16$

$x^2 - 25 = 0$

$(x+5)(x-5) = 0$

$x = -5$ not a solution.

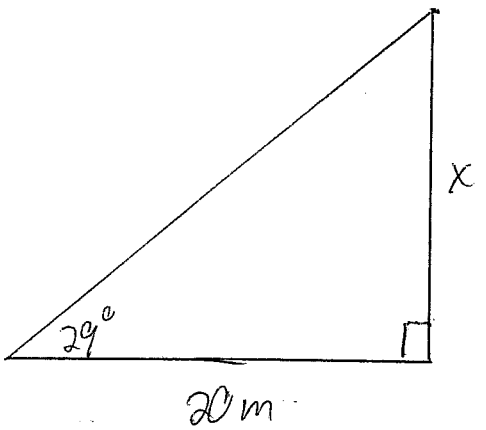
$x = 5$

Question 6. *(4 marks)* A tree casts a 20m long shadow when the angle of elevation of the sun is 29°. How tall is the tree?

$\tan 29^\circ = \frac{x}{20}$

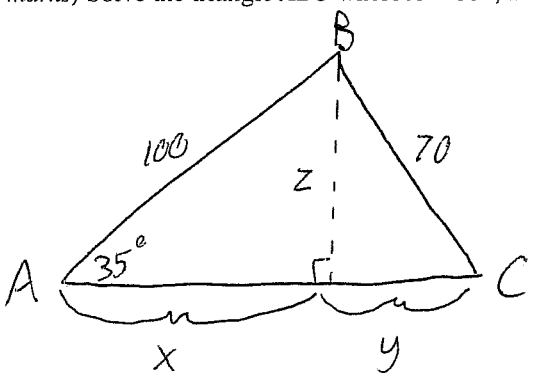
$20\tan 29^\circ = x$

$x \doteq 11.09$



$\therefore$ the tree is 11.09 m tall

Bonus. *(3 marks)* Solve the triangle ABC where $A = 35°$, $a = 70$, $c = 100$. *Do not assume that an angle of the triangle is* 90°.



$\sin 35^\circ = \frac{z}{100}$

$z \doteq 57.41$

$\cos 35 \doteq \frac{x}{100}$

$x \doteq 81.9$

$70^2 = z^2 + y^2$

$y = \sqrt{70^2 - (57.4)^2}$

$= 40$

$\sin C = \frac{z}{70}$

$C = \sin^{-1}\left(\frac{57.4}{70}\right) \doteq 55^\circ$

$B = 180^\circ - 35^\circ - 55^\circ = 90^\circ$

$\therefore b = 81.9 + 40 = 122$