(21) $$\frac{x-4+3x^3}{4+x}$$

(22) $$\frac{x^4-6x^2+5x+4}{x-2}$$

(23) $$\frac{x^4+4x^3-5x^2-12x+6}{x^2-3}$$

(24) $$\frac{x^4+2x^3+2x^2-x-1}{x^2+1}$$

(25) $$\frac{x^4-5x^2+4}{x^2-1}$$

(26) $$\frac{x^5-2x^3+5x+1}{x^2-x+2}$$

(27) $$\frac{x^3-x^2+x+3}{x^2-2x+3}$$

(28) $$\frac{3x^4-10x^2-2x+2}{x^2+2x+1}$$

(29) $$\frac{6x^3+7x^2-18x+15}{2x^2+3x+5}$$

(30) $$\frac{x^5-2x^3-3x^2+9}{x^2-2}$$

(31) $$\frac{36x^4+72x^3-121x^2-142x+120}{6x^2+11x-10}$$

(32) $$\frac{12x^6+11x^5+3x^4+10x^3-9x^2+3x-6}{4x^4+5x^3-3}$$

ANSWERS

(1) $4x+3$ and 0 (2) $x+2$ and -12 (3) $x+1$ and -16 (4) $x+1$ and -1 (5) $3x+2$ and 0

(6) $3x+7$ and 6 (7) x^2-x-6 and 4 (8) $3x^2-4x+6$ and 0 (9) $5x^2-x+6$ and 0

(10) $5x^2+2x-3$ and 3 (11) x^2-x+2 and 0 (12) $2x^2+5x+2$ and 8

(13) $3x^2+7$ and 0 (14) $6x^2+18x+78$ and 313 (15) x^3-3x^2+3x-1 and 0

(16) $x^3-3x^2+8x-12$ and 15 (17) $x^2+5x+15$ and 41 (18) $2x^2+4x+9$ and 0

(19) $5x^2-4x+4$ and 0 (20) $2x^2+9x+18$ and 35 (21) $3x^2-12x+49$ and -200

(22) x^3+2x^2-2x+1 and 6 (23) x^2+4x-2 and 0 (24) x^2+2x+1 and $-3x-2$

(25) x^2-4 and 0 (26) x^3+x^2-3x-5 and $6x+11$

(27) $x+1$ and 0 (28) $3x^2-6x-1$ and $6x+3$ (29) $3x-1$ and $-30x+20$

(30) x^3-3 and 3 (31) $6x^2+x-12$ and 0 (32) $3x^2-x+2$ and 0

FACTORING

GREATEST COMMON FACTOR

$$ax + ab = a(x+b)$$

TRINOMIALS (with coefficient of $x^2 = 1$)

$$x^2 + (a+b)x + ab = (x+a)(x+b)$$

TRINOMIALS (with coefficient of $x^2 \neq 1$)

$$acx^2 + (ad+bc)x + bd = (ax+b)(cx+d)$$

SPECIAL FACTORIZATIONS

PERFECT SQUARE TRINOMIALS

$$x^2 + 2xy + y^2 = (x+y)^2$$

$$x^2 - 2xy + y^2 = (x-y)^2$$

DIFFERENCE OF SQUARES

$$x^2 - y^2 = (x+y)(x-y)$$

DIFFERENCE AND SUM OF CUBES

$$x^3 - y^3 = (x-y)(x^2+xy+y^2)$$

$$x^3 + y^3 = (x+y)(x^2-xy+y^2)$$

NOTE: a TRINOMIAL $ax^2 + bx + c$ can be factored over the integers only if $b^2 - 4ac = 0, 1, 4, 9, 16, \cdots$.

also, if $b^2 - 4ac = 0$, it is a perfect square trinomial.

Factor each polynomial:

Greatest Common Factor:

$$5x^2 - 10x = 5x(x-2)$$

check: multiply $5x(x-2) = 5x^2 - 10x$ ✓

Common Factors in Factoring by Grouping:

$$2x^2 - 2x + 3x - 3 = (2x^2 - 2x) + (3x - 3) \quad , \text{ grouping}$$
$$= 2x(x-1) + 3(x-1) \quad , \text{ common factors}$$
$$= (x-1)(2x+3) \quad , \text{ common binomial factor}$$

check: multiply $(x-1)(2x+3) = 2x^2 + 3x - 2x - 3$
$$= 2x^2 - 2x + 3x - 3 \text{ ✓}$$

Trinomials (with coefficient of $x^2 = 1$):

$$x^2 + 2x - 8 = (x+a)(x+b) \text{ where } ab = -8 \text{ and } (a+b) = 2$$
$$= (x+4)(x-2)$$

check: multiply $(x+4)(x-2) = x^2 + 2x - 8$ ✓

Trinomials (with coefficient of $x^2 \neq 1$):

We factor this form by grouping as follows:

$2x^2 + 3x - 9$, consider the product $(2x^2)(-9) = -18x^2 = (6x)(-3x)$
such that $6x - 3x = 3x$ and write

$$2x^2 + 3x - 9 = 2x^2 + 6x - 3x - 9$$
$$= 2x(x+3) - 3(x+3) \quad , \text{ factor by}$$
$$= (x+3)(2x-3) \qquad \text{grouping}$$

check: multiply $(x+3)(2x-3) = 2x^2 + 3x - 9$ ✓

Note: If necessary, we can factor the other trinomial forms by grouping also.

FACTORING - EXAMPLES

Perfect Square Trinomials:

$$x^2 + 2x + 1 = x^2 + 2 \cdot x \cdot 1 + 1^2 \text{ , hence}$$
$$= (x+1)^2$$

$$4x^2 - 12x + 9 = (2x)^2 - 2(2x) \cdot 3 + 3^2 \text{ , hence}$$
$$= (2x-3)^2$$

Difference of Squares:

$$x^2 - 4 = x^2 - 2^2 \text{ , hence}$$
$$= (x+2)(x-2)$$

Difference and Sum of Cubes:

$$x^3 - 8 = x^3 - 2^3 \text{ , hence}$$
$$= (x-2)(x^2 + 2x + 2^2)$$
$$= (x-2)(x^2 + 2x + 4)$$

$$8x^3 + 27 = (2x)^3 + 3^3 \text{ , hence}$$
$$= (2x+3)((2x)^2 - (2x)3 + 3^2)$$
$$= (2x+3)(4x^2 - 6x + 9)$$

Greatest Common Factoring as a _first step_:

$$2x^3 - 14x^2 + 36 = 2x(x^2 - 7x + 18) \text{ , greatest common factor}$$
$$= 2x(x-9)(x+2)$$

Note: Not all trinomials are factorable over the integers.
Consider $2x^2 + 3x - 1$, then $b^2 - 4ac = 3^2 - 4(2)(-1) = 9 + 8 = 17$,
hence, it is not factorable since 17 is not a perfect square integer.

FACTORING — EXERCISES

① Factor out the greatest common factor:

   ⓐ $10x + 20$ ⓑ $18x^3 - 9x$

   ⓒ $28x^5 + 14x^4 - 21x^3$ ⓓ $50x^2y^2 - 10xy^2$

   ⓔ $2x^3y - 6x^2y^2 + 14xy^3$ ⓕ $15x^5 - 18x^4 + 21x^3 - 48x^2$

   ⓖ $56x^5y^4 + 21x^3y^2 - 35x^2y^3 - 49x^4y^5$

   ⓗ $x(x+5) + 4(x+5)$

   ⓘ $2x(x-1) - 3(x-1)$

   ⓙ $7x^2(x+1)^2 + 8x(x+1)^2$

② Factor by grouping:

   ⓐ $x^2 + 3x + 2x + 6$ ⓑ $x^2 - 5x + 4x - 20$

   ⓒ $x^2 + 7x - 2x - 14$ ⓓ $2x^2 + 10x + 7x + 35$

   ⓔ $3x^2 - 9x - 8x + 24$ ⓕ $5x^2 - 10x - x + 2$

   ⓖ $4x^2 + 10x - 6x - 15$ ⓗ $x - 1 + xy - y$

   ⓘ $3xy - y^2 + 3x - y$ ⓙ $3x^3 + 3x^2 - 2x - 2$

③ Factor each trinomial:

   ⓐ $x^2 + 4x + 3$ ⓑ $x^2 + 10x - 11$

   ⓒ $x^2 + x - 20$ ⓓ $x^2 - 13x + 42$

   ⓔ $x^2 + 5x - 36$ ⓕ $x^2 - 2x - 63$

③ ⑧ $x^2 - 9x + 20$ ⓗ $x^2 - 21x - 100$

  ⓘ $x^2 - 25x + 126$ ⓙ $x^2 + 8x - 105$

④ Factor each trinomial:

  ⓐ $3x^2 + 8x + 5$ ⓑ $2x^2 + 5x - 3$

  ⓒ $5x^2 - 7x - 6$ ⓓ $6x^2 + 7x - 10$

  ⓔ $4x^2 - 12x + 5$ ⓕ $2x^2 - x - 6$

  ⓖ $8x^2 + 14x + 5$ ⓗ $7x^2 - 27x - 4$

  ⓘ $12x^2 + 8x - 15$ ⓙ $21x^2 + 25x - 4$

  ⓚ $2x^2 + 5x - 18$ ⓛ $10x^2 - 23x + 12$

  ⓜ $20x^2 - 39x - 11$ ⓝ $18x^2 - 9x - 5$

⑤ Factor each perfect square trinomial:

  ⓐ $x^2 + 10x + 25$ ⓑ $x^2 - 2x + 1$

  ⓒ $x^2 - 22x + 121$ ⓓ $4x^2 + 20x + 25$

  ⓔ $16x^2 - 56x + 49$ ⓕ $36x^2 - 60x + 25$

  ⓖ $25x^2 + 10x + 1$ ⓗ $9x^2 - 24x + 16$

  ⓘ $1 - 4x + 4x^2$ ⓙ $81x^2 + 180 + 100$

⑥ Factor each difference of squares:

  ⓐ $x^2 - 25$ ⓑ $x^2 - 49$

FACTORING - EXERCISES

⑥ ⓒ $4x^2 - 81$

ⓔ $49x^2 - 36y^2$

ⓖ $49 - 9x^2$

ⓘ $64x^2 - 100$

ⓓ $16x^2 - 1$

ⓕ $9x^2 - 64y^2$

ⓗ $16x^2 - 121y^2$

ⓙ $x^4 - 1$

⑦ Factor each difference or sum of cubes:

ⓐ $x^3 - 1$

ⓒ $x^3 - 125$

ⓔ $8x^3 - 27y^3$

ⓖ $27x^3 - 125$

ⓘ $512x^3 - 343$

ⓑ $x^3 + 27$

ⓓ $x^3 + 64$

ⓕ $64x^3 + 27$

ⓗ $8x^3 + 729y^3$

ⓙ $125x^3 + 1000$

⑧ Factor completely:

ⓐ $27x^3 - 15x$

ⓒ $2x^6 + 8x^5 - 42x^4$

ⓔ $16x^5 + 48x^4 + 36x^3$

ⓖ $10x^3 - 270$

ⓙ $12x^5 + 12x^3 - 4x^4 - 4x^2$

ⓚ $54x^4 + 2000x$

ⓜ $(x-2)^2 + 3(x-2)$

ⓑ $50x^3 - 100x^2 - 10x^2 + 20x$

ⓓ $15x^4 - 25x^3 + 10x^2$

ⓕ $3x^3 - 24x^2 + 48x$

ⓗ $16ax^3 + 54ay^3$

ⓘ $x^6 - 64$

ⓛ $x^3 - 3x^2 - 4x + 12$

ⓝ $x^2(x-2) - (x-2)$

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FACTORING — EXERCISES (ANSWERS)

⑧ ⓞ $(5x+7)^2 - 16$ ⓟ $7x^4 + 7x^3 - 140x^2$ ⓠ $25x^3 + 65x^2 - 30x$

ⓡ $x^3 - 3x^2 - 4x + 12$ ⓢ $120x^5 + 110x^4 - 50x^3$ ⓣ $(x+1)^2 - (x+1) - 6$

ⓤ $(x^2-9)^2 + 8x(x^2-9)$ ⓥ $(x-1)^3 - 8$

⑨ Determine whether each trinomial is factorable over the integers or not.

ⓐ $x^2 + 5x - 3$ ⓑ $x^2 + 3x - 88$ ⓒ $3x^2 - 15x + 16$

ⓓ $5x^2 + 13x - 6$ ⓔ $2x^2 + 5x - 5$ ⓕ $9x^2 - 3x - 2$

ANSWERS

① ⓐ $10(x+2)$ ⓑ $9x(2x^2-1)$ ⓒ $7x^3(4x^2+2x-3)$ ⓓ $10xy^2(5xy-1)$ ⓔ $2xy(x^2-3xy+7y^2)$
ⓕ $3x^2(5x^3-6x^2+7x-16)$ ⓖ $7x^2y^2(8x^3y^2+3x-5y-7x^2y^3)$ ⓗ $(x+5)(x+4)$
ⓘ $(x-1)(2x-3)$ ⓙ $x(x+1)^2(7x+8)$

② ⓐ $(x+3)(x+2)$ ⓑ $(x-5)(x+4)$ ⓒ $(x+7)(x-2)$ ⓓ $(x+5)(2x+7)$ ⓔ $(x-3)(3x-8)$ ⓕ $(x-2)(5x-1)$
ⓖ $(2x+5)(2x-3)$ ⓗ $(x-1)(1+y)$ ⓘ $(y+1)(3x-y)$ ⓙ $(x+1)(3x^2-2)$

③ ⓐ $(x+1)(x+3)$ ⓑ $(x+11)(x-1)$ ⓒ $(x+5)(x-4)$ ⓓ $(x-6)(x-7)$ ⓔ $(x+9)(x-4)$ ⓕ $(x-9)(x+7)$
ⓖ $(x-4)(x-5)$ ⓗ $(x-25)(x+4)$ ⓘ $(x-7)(x-18)$ ⓙ $(x+15)(x-7)$

④ ⓐ $(x+1)(3x+5)$ ⓑ $(x+3)(2x-1)$ ⓒ $(x-2)(5x+3)$ ⓓ $(x+2)(6x-5)$ ⓔ $(2x-5)(2x+7)$ ⓕ $(x-2)(2x+3)$
ⓖ $(4x+5)(2x+1)$ ⓗ $(7x+1)(x-4)$ ⓘ $(6x-5)(2x+3)$ ⓙ $(7x-1)(3x+4)$ ⓚ $(x-2)(2x+9)$
ⓛ $(5x-4)(2x-3)$ ⓜ $(4x+1)(5x-11)$ ⓝ $(3x+1)(6x-5)$

⑤ ⓐ $(x+5)^2$ ⓑ $(x-1)^2$ ⓒ $(x-11)^2$ ⓓ $(2x+5)^2$ ⓔ $(4x-7)^2$ ⓕ $(6x-5)^2$ ⓖ $(5x+1)^2$ ⓗ $(3x-4)^2$
ⓘ $(1-2x)^2$ ⓙ $(9x+10)^2$

⑥ ⓐ $(x+5)(x-5)$ ⓑ $(x+7)(x-7)$ ⓒ $(2x+9)(2x-9)$ ⓓ $(4x+1)(4x-1)$ ⓔ $(7x+6y)(7x-6y)$
ⓕ $(3x+8y)(3x-8y)$ ⓖ $(7+3x)(7-3x)$ ⓗ $(4x+11y)(4x-11y)$ ⓘ $(8x+10)(8x-10)$ ⓙ $(x^2+1)(x+1)(x-1)$

⑦ ⓐ $(x-1)(x^2+x+1)$ ⓑ $(x+3)(x^2-3x+9)$ ⓒ $(x-5)(x^2+5x+25)$ ⓓ $(x+4)(x^2-4x+16)$ ⓔ $(2x-3y)(4x^2+6xy+9y^2)$
ⓕ $(4x+3)(16x^2-12x+9)$ ⓖ $(3x-5)(9x^2+15x+25)$ ⓗ $(2x+9y)(4x^2-18xy+81y^2)$
ⓘ $(8x-7)(64x^2+56x+49)$ ⓙ $(5x+10)(25x^2-50x+100)$

⑧ ⓐ $3x(9x^2-5)$ ⓑ $10x(x-2)(5x-1)$ ⓒ $2x^4(x+7)(x+3)$ ⓓ $5x^2(x-1)(3x-2)$ ⓔ $4x^3(2x+3)^2$ ⓕ $3x(x-4)^2$
ⓖ $10(x-3)(x^2+3x+9)$ ⓗ $2a(2x+3y)(4x^2-16xy+9y^2)$ ⓘ $4x^2(x^2+1)(3x-1)$ ⓙ $(x+2)(x-2)(x^4+4x^2+16)$
ⓚ $2x(3x+10)(9x^2-30x+100)$ ⓛ $(x-3)(x+2)(x-2)$ ⓜ $(x-2)(x+1)$ ⓝ $(x-2)(x+1)(x-1)$ ⓞ $(5x+3)(5x+11)$
ⓟ $7x^2(x+5)(x-4)$ ⓠ $5x(5x-2)(x+3)$ ⓡ $(x-3)(x+2)(x-2)$ ⓢ $10x^3(3x-1)(4x+5)$
ⓣ $(x-2)(x+3)$ ⓤ $(x+3)(x-3)(x+9)(x-1)$ ⓥ $(x-3)(x^2+3)$

⑨ ⓐ No ⓑ Yes ⓒ No ⓓ Yes ⓔ No ⓕ Yes

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