INTEGER EXPONENTS IN ALGEBRA

THE DEFINITION OF AN EXPONENTIAL EXPRESSION

a^n is an exponential expression where a is the BASE and n is the EXPONENT (or POWER) such that:

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}$$

$$a^0 = 1, \quad a \neq 0$$

$$a^{-n} = \frac{1}{a^n}, \quad a \neq 0$$

NOTE: a^n is read "a to the n^{th} power", $a^1 = a$, a^2 is read "a squared", and a^3 is read "a cubed".

THE RULES OF EXPONENTS

PRODUCT	$a^m \cdot a^n = a^{m+n}$
POWER	$(a^m)^n = a^{mn}$
QUOTIENT	$\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$

PROPERTIES

① $(ab)^n = a^n b^n$

② $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

③ $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$

④ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

INTEGER EXPONENTS IN ALGEBRA – EXAMPLES

Simplify, expressing the answers with positive exponents only.

① $\dfrac{a^2 \cdot a^3}{(a^2)^3} = \dfrac{a^{2+3}}{a^{2\cdot 3}} = \dfrac{a^5}{a^6}$ ← Product Rule ← Power Rule

$= a^{5-6}$ Quotient Rule

$= a^{-1}$

$= \dfrac{1}{a^1}$ Definition

$= \dfrac{1}{a}$ (Definition)

② $\left(\dfrac{2a^2}{3b^3}\right)^2 = \dfrac{(2a^2)^2}{(3b^3)^2}$ Property ②

$= \dfrac{2^2(a^2)^2}{3^2(b^3)^2}$ } Property ①

$= \dfrac{4a^4}{9b^6}$ Power Rule

③ $\dfrac{(3x^2)^{-2}}{(2y^3)^{-3}} = \dfrac{(2y^3)^3}{(3x^2)^2}$ Property ③

$= \dfrac{2^3(y^3)^3}{3^2(x^2)^2}$ } Property ①

$= \dfrac{8y^9}{9x^4}$ Power Rule

④ $\left(\dfrac{x^{-2}}{2y^3}\right)^{-4} = \left(\dfrac{2y^3}{x^{-2}}\right)^4$ Property ④

$= \dfrac{(2y^3)^4}{(x^{-2})^4}$ Property ②

$= \dfrac{16y^{12}}{x^{-8}}$ Power Rule

$\dfrac{16y^{12}}{\frac{1}{x^8}} = 16y^{12}x^8$ (Definition)

INTEGER EXPONENTS IN ALGEBRA — EXERCISES

Simplify, expressing the answers with positive exponents only:

(1) $a^2 \cdot a^3$

(2) $(a^2)^3$

(3) $\frac{a^5}{a^2}$

(4) $x^2 \cdot x^3 \cdot x^4$

(5) $\frac{x^7 x^9}{(x^3)^2}$

(6) $(2x^3)(7x^2)$

(7) $6x^2(2x^4)^2$

(8) $\frac{9x^5}{3x^2}$

(9) $(-2x^3)^4$

(10) $x^{-3} \cdot x^5 \cdot x^{-4}$

(11) $(-3x)^3(-3x)$

(12) $(-2x^2)(4x^3)^{-1}$

(13) $(x^2)^3 (x^3)^5 (x^4)^{-2}$

(14) $(-2x^2y^3)^5$

(15) $(-5a^2b^3)(2a^{-2}b^{-4})$

(16) $\frac{(-6x^{15})(-2x^0)^2}{(-2x^4)^3}$

(17) $\left(\frac{5x^2}{y^{-2}}\right)^{-4}$

(18) $\frac{x^0 y^{-2} z^3}{(xy^{-1}z^{-3})^{-1}}$

(19) $\left(\frac{3a^2b^{-2}}{ab^2}\right)^{-2}$

(20) $\frac{a^{-3}}{b^{-3}}\left(\frac{a}{b}\right)^3$

(21) $\left(\frac{x^{-2}}{2y^{-1}}\right)^2$

(22) $\frac{(-x^4y^{-2}z^2)^0}{-(x^4y^{-2}z^2)^{-2}}$

(23) $\left(\frac{4x^{-1}y^{-40}}{2^{-2}x^4y^{-10}}\right)^{-2}$

(24) $\left(\frac{3^{-4}x^{-3}}{3^{-3}x^{-6}}\right)^{-2}$

(25) $\frac{(2xy^{-3})^{-2}}{(3x^{-2}y^4)^{-3}}$

(26) $\frac{(xy)^{-3}(xy)^5}{(xy)^{-4}}$

(27) $\left[\frac{(a^{55}b^{23})^3}{a^{-16}}\right]^0$

(28) $\frac{(a^{-2}b^3)^{-4}}{(a^{-3}b^2)^{-2}(ab)^{-4}}$

(29) $\frac{x^{11}(x^2)^4}{(x^3)^3(x^2)^{-6}}$

(30) $\frac{(-2xy^{-3})^{-3}}{(xy^{-1})^{-1}}$

INTEGER EXPONENTS IN ALGEBRA – EXERCISES (ANSWERS)

(1) a^5 (2) a^6 (3) a^3 (4) x^9 (5) x^{10} (6) $14x^5$

(7) $24x^{10}$ (8) $3x^3$ (9) $16x^{12}$ (10) $\frac{1}{x^2}$ (11) $81x^4$ (12) $-\frac{1}{2x}$

(13) x^{13} (14) $-32x^{10}y^{15}$ (15) $-\frac{10}{b}$ (16) $3x^3$ (17) $\frac{1}{625x^8y^8}$

(18) $\frac{x}{y^3}$ (19) $\frac{b^8}{9a^2}$ (20) 1 (21) $\frac{y^2}{4x^4}$ (22) $-\frac{x^8z^4}{y^4}$

(23) $\frac{x^{10}y^{60}}{256}$ (24) $\frac{9}{x^6}$ (25) $\frac{27y^{18}}{4x^8}$ (26) x^6y^6 (27) 1

(28) $\frac{a^6}{b^4}$ (29) x^{22} (30) $-\frac{y^8}{8x^2}$