POLYNOMIALS I

THE DEFINITION OF A POLYNOMIAL

A POLYNOMIAL in x is a sum of terms that may be denoted, in descending powers of x, as follows:

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

The DEGREE of the polynomial is the non-negative integer n.
The COEFFICIENTS of the polynomial are the real numbers $a_n, a_{n-1}, \cdots, a_1, a_0$

NOTE: A polynomial with 1 term is called a MONOMIAL.
A polynomial with 2 terms is called a BINOMIAL.
A polynomial with 3 terms is called a TRINOMIAL.

EXAMPLE: $3x^2 - 5x + 1$ is a trinomial of degree 2 with the coefficients 3, -5, and 1.

ADDING AND SUBTRACTING POLYNOMIALS

COMBINE LIKE TERMS

NOTE: Like terms have the same variable(s) and exponent(s).

MULTIPLYING POLYNOMIALS

$$a(c+d) = ac + ad$$

$$(a+b)(c+d) = a(c+d) + b(c+d)$$

SPECIAL BINOMIAL PRODUCTS

SQUARES OF A BINOMIAL

$$(x+y)^2 = x^2 + 2xy + y^2 \text{ and } (x-y)^2 = x^2 - 2xy + y^2$$

NOTE: These products are perfect square trinomials

PRODUCT OF A BINOMIAL SUM AND DIFFERENCE

$$(x+y)(x-y) = x^2 - y^2$$

NOTE: This product is a difference of squares

$$(a+b)(c+d+e) = a(c+d+e) + b(c+d+e)$$

POLYNOMIALS I – EXAMPLES

(1) Add or subtract as indicated:

$$(5x^2 - x + 3) - (2x^2 - 7x + 6) = 5x^2 - x + 3 - 2x^2 + 7x - 6$$
$$= 3x^2 + 6x - 3$$

(2) Multiply and simplify:

$$3x^2(x - 2) = 3x^2 \cdot x + 3x^2(-2) = 3x^3 - 6x^2$$

(3) Multiply and simplify:

$$(x+5)(x-3) = x(x-3) + 5(x-3) = x^2 - 3x + 5x - 15$$
$$= x^2 + 2x - 15$$

MULTIPLYING 2 BINOMIALS BY THE FOIL METHOD

EXAMPLE: $(x+5)(x-3) = \underbrace{x^2}_{F} - \underbrace{3x}_{O} + \underbrace{5x}_{I} - \underbrace{15}_{L} = x^2 + 2x - 15$

- F → FIRST TERMS PRODUCT
- O → OUTER TERMS PRODUCT
- I → INNER TERMS PRODUCT
- L → LAST TERMS PRODUCT

(4) Find the special products and simplify:

$$(x+3)^2 = x^2 + 2 \cdot x \cdot 3 + 3^2 = x^2 + 6x + 9$$

$$(2x-5)^2 = (2x)^2 - 2 \cdot 2x \cdot 5 + 5^2 = 4x^2 - 20x + 25$$

$$(3x+4)(3x-4) = (3x)^2 - 4^2 = 9x^2 - 16$$

(5) Multiply and simplify:

$$(a+b)(a^2 - ab + b^2) = a(a^2 - ab + b^2) + b(a^2 - ab + b^2)$$
$$= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$$
$$= a^3 + b^3 \text{, a } \underline{\text{sum of cubes}}.$$

Similarly,

$$(a-b)(a^2 + ab + b^2) = a^3 - b^3 \text{, a } \underline{\text{difference of cubes}}.$$

POLYNOMIALS I - EXERCISES

(1) For each polynomial, give its degree and name (if applicable):

(a) $4x^5$

(b) $3x^2+5$

(c) $7x^2-4x+6$

(d) x^3+2x^2+3x-4

→ (e) $25x^2-16$

(2) Add or subtract as indicated:

(a) $3x^2+5x^2$

(b) $(5x+4)+(2x-3)$

(c) $(7x^2-6x+5)+(x^2+4x-2)$

(d) $(6x+5)-(3x+1)$

(e) $(5x^2+4x-6)-(x^2-2x+1)$

(f) $(25x^3+14x)-(20x^3-8x^2+9x+1)$

(g) $(7x^4+3x^2+2x)-(18x^4-5x^2+x)$

(h) $(17x^4-11x^2-10x+6)+(-7x^4+11x^2+10x-15)$

(i) $(9x^2+2x-1)-(11x^2+5x-8)+(2x^2+4x-7)$

(j) $(2x^2+x-5)-(3x^2+2x-2)+(x^2+x+3)$

(3) Multiply and simplify:

(a) $x(x+2)$

(b) $3x(5x-8)$

(c) $x^3(x+12)$

(d) $4x^2(x-7)$

POLYNOMIALS I - EXERCISES

③ (e) $-2x^3(x^2-5x)$ → (f) $x^2(3x^2-x+6)$

(g) $10(5x^2+7x-4)$ (h) $-4x^2(3x^3-12x^2-6)$

(i) $5x^4(x^3-2x^2-3x-4)$ → (j) $-9x^5(-3x^6-2x^4+8x^2)$

④ Multiply and simplify (you may use FOIL):

(a) $(x+4)(x+7)$ (b) $(x-5)(x+2)$

(c) $(2x-1)(x+3)$ (d) $(3x+4)(4x+3)$

(e) $(2x+5)(5x-7)$ (f) $(7x+2)(3x-4)$

(g) $(4x-1)(6x-5)$ (h) $(2x-7)(5x+3)$

(i) $x(2x-5)(x+3)$ (j) $3x^3(x-5)(2x+3)$

⑤ Find the special products and simplify:

(a) $(x+1)^2$ (b) $(x-1)^2$

(c) $(2x+5)^2$ (d) $(4x-3)^2$

(e) $(x+1)(x-1)$ (f) $(x+5)(x-5)$

(g) $(4x+3)(4x-3)$ (h) $(5x+4)(5x-4)$

(i) $x^2(3x-1)^2$ (j) $7x^3(3x+2)(3x-2)$

⑥ Multiply and simplify:

(a) $(x-3)(x^2+2x-1)$ (b) $(x+1)(x^2-5x+3)$

(c) $(x+3)(2x^2-4x+3)$ (d) $(2x-7)(x^2-6x+1)$

POLYNOMIALS I – EXERCISES (ANSWERS)

(6) (e) $(x-1)(x^2+x+1)$
(f) $(2x+3)(4x^2-6x+9)$
(g) $10(4x-1)(16x^2+4x+1)$
(h) $3x^3(2x^2+5x)(x^3+2x+1)$
(i) $(4x+3)(5x^3-4x^2+x-5)$
(j) $(x^2+2x+1)(3x^2-6x-1)$

(7) Simplify:
(a) $(x+y)^3$ (b) $(x-y)^3$ (c) $(x+1)^3$ (d) $(2x-3)^3$
(e) $4x^2(x+1)-2x(x^2+2x)$
(f) $2(x-3)^2+3(x+3)(x-3)$
(g) $5(x^2+y^2)(x+y)(x-y)$
(h) $-2x^3(-x^2-2x+3)(2x-1)$
(i) $(x-1)^3-(x+1)^2+4(x+1)(x-1)$
(j) $(x+2)(x^2-2x+4)(x-2)(x^2+2x+4)$

ANSWERS

(1) (a) 5, monomial (b) 2, binomial (c) 2, trinomial (d) 3, polynomial (e) 4, binomial

(2) (a) $8x^2$ (b) $7x+1$ (c) $8x^2-2x+3$ (d) $3x+4$ (e) $4x^2+6x-7$ (f) $5x^3+8x^2+5x-1$
(g) $-11x^4+8x^2+x$ (h) $10x^4-9$ (i) x (j) 0

(3) (a) x^2+2x (b) $15x^2-24x$ (c) x^4+12x^3 (d) $4x^3-28x^2$ (e) $-2x^5+10x^4$ (f) $3x^4-x^3+6x^2$
(g) $50x^2+70x-40$ (h) $-12x^5+48x^4+24x^2$ (i) $5x^7-10x^6-15x^5-20x^4$
(j) $27x^{11}+18x^9-72x^7$

(4) (a) $x^2+11x+28$ (b) $x^2-3x-10$ (c) $2x^2+5x-3$ (d) $12x^2+25x+12$ (e) $10x^2+11x-35$
(f) $21x^2-22x-8$ (g) $24x^2-26x+5$ (h) $10x^2-29x-21$ (i) $2x^3+x^2-15x$ (j) $6x^5-21x^4-45x^3$

(5) (a) x^2+2x+1 (b) x^2-2x+1 (c) $4x^2+20x+25$ (d) $16x^2-24x+9$ (e) x^2-1 (f) x^2-25
(g) $16x^2-9$ (h) $25x^2-16$ (i) $9x^4-6x^3+x^2$ (j) $63x^5-28x^3$

(6) (a) x^3-x^2-7x+3 (b) x^3-4x^2-2x+3 (c) $2x^3+2x^2-9x+9$ (d) $2x^3-19x^2+44x-7$
(e) x^3-1 (f) $8x^3+27$ (g) $640x^3-10$ (h) $6x^8+15x^7+12x^6+36x^5+15x^4$
(i) $20x^4-x^3-8x^2-17x-15$ (j) $3x^4-10x^2-8x-1$

(7) (a) $x^3+3x^2y+3xy^2+y^3$ (b) $x^3-3x^2y+3xy^2-y^3$ (c) x^3+3x^2+3x+1
(d) $8x^3-36x^2+54x-27$ (e) $2x^3$ (f) $5x^2-12x-9$ (g) $5x^4-5y^4$
(h) $4x^6+6x^5-16x^4+6x^3$ (i) x^3+x-6 (j) x^6-64

POLYNOMIALS II

DIVIDING POLYNOMIALS

ILLUSTRATIVE EXAMPLE

Divide $\frac{3x^2+4x-3}{x+2}$ by LONG DIVISION as follows:

1 $\frac{3x^2}{x}$ $\frac{-2x}{x}$ 3

$$x+2 \;\overline{)\; 3x^2+4x-3 \;}\;\; 3x-2$$

$$3x^2+6x$$

2 $\frac{3x(x+2)}{\text{SUBTRACT}}$

$$-2x-3$$

$$-2x-4$$

$$1$$

$\frac{-2(x+2)}{\text{SUBTRACT}}$ 4

check: $\underbrace{3x^2+4x-3}_{\text{DIVIDEND}} = \underbrace{(x+2)}_{\text{DIVISOR}} \cdot \underbrace{(3x-2)}_{\text{QUOTIENT}} + \underbrace{1}_{\text{REMAINDER}}$?

$= 3x^2-2x+6x-4+1$?

$= 3x^2+4x-3$ true

hence, $\boxed{\frac{3x^2+4x-3}{x+2} = 3x-2+\frac{1}{x+2}}$

NOTE: ① In long division, both the dividend and the divisor must be written in descending powers of X.

② Long division is complete when the degree of the remainder is less than the degree of the divisor.

POLYNOMIALS II – EXAMPLES

NOTE: In either the dividend or the divisor, any missing terms in descending powers of x, must be entered with a 0 coefficient to keep like terms in the same column.

(1) Divide $\dfrac{3x^3 - 5x + 2}{x - 1}$

then

$$
\begin{array}{r}
3x^2 + 3x - 2 \quad \leftarrow \text{Quotient} \\
x - 1 \;\overline{)\; 3x^3 + 0x^2 - 5x + 2} \\
\underline{3x^3 - 3x^2} \\
3x^2 - 5x \\
\underline{3x^2 - 3x} \\
-2x + 2 \\
\underline{-2x + 2} \\
0 \quad \leftarrow \text{Remainder}
\end{array}
$$

(2) Divide $\dfrac{2x^4 - x^3 + 3x^2 + x + 2}{x^2 + 1}$

then

$$
\begin{array}{r}
2x^2 - x + 1 \quad \leftarrow \text{Quotient} \\
x^2 + 0x + 1 \;\overline{)\; 2x^4 - x^3 + 3x^2 + x + 2} \\
\underline{2x^4 + 0x^3 + 2x^2} \\
-x^3 + x^2 + x \\
\underline{-x^3 + 0x^2 - x} \\
x^2 + 2x + 2 \\
\underline{x^2 + 0x + 1} \\
2x + 1 \quad \leftarrow \text{Remainder}
\end{array}
$$

POLYNOMIALS II - EXERCISES

Divide by long division to find the quotient and remainder:

(1) $\frac{4x^2+7x+3}{x+1}$

(2) $\frac{x^2+7x-2}{x+5}$

(3) $\frac{x^2-3x-20}{x-4}$

(4) $\frac{x^2-x-3}{x-2}$

(5) $\frac{6x^2+x-2}{2x-1}$

(6) $\frac{4x+3x^2-1}{x-1}$

(7) $\frac{x^3-2x^2-5x+10}{x-1}$

(8) $\frac{3x^3+5x^2-6x+18}{x+3}$

(9) $\frac{5x^3-11x^2+8x-12}{x-2}$

(10) $\frac{5x^3+12x^2+x-3}{x+2}$

(11) $\frac{3x-x^2+2x^3+2}{2x+1}$

(12) $\frac{4x^3+8x^2-x+6}{2x-1}$

(13) $\frac{6x^3-3x^2+14x-7}{2x-1}$

(14) $\frac{6x^3-5x^2+2x+1}{-4+x}$

(15) $\frac{x^4-4x^3+6x^2-4x+1}{x-1}$

(16) $\frac{x^4-2x^3+5x^2-4x+3}{x+1}$

(17) $\frac{x^3+2x^2-4}{x-3}$

(18) $\frac{2x^3+x-18}{x-2}$

(19) $\frac{5x^3+x^2+4}{x+1}$

(20) $\frac{2x^3+5x^2-1}{x-2}$

POLYNOMIALS II – EXERCISES

(21) $\dfrac{x-4+3x^3}{4+x}$

(22) $\dfrac{x^4-6x^2+5x+4}{x-2}$

(23) $\dfrac{x^4+4x^3-5x^2-12x+6}{x^2-3}$

(24) $\dfrac{x^4+2x^3+2x^2-x-1}{x^2+1}$

(25) $\dfrac{x^4-5x^2+4}{x^2-1}$

(26) $\dfrac{x^5-2x^3+5x+1}{x^2-x+2}$

(27) $\dfrac{x^3-x^2+x+3}{x^2-2x+3}$

(28) $\dfrac{3x^4-10x^2-2x+2}{x^2+2x+1}$

(29) $\dfrac{6x^3+7x^2-18x+15}{2x^2+3x+5}$

(30) $\dfrac{x^5-2x^3-3x^2+9}{x^2-2}$

(31) $\dfrac{36x^4+72x^3-121x^2-142x+120}{6x^2+11x-10}$

(32) $\dfrac{12x^6+11x^5+3x^4+10x^3-9x^2+3x-6}{4x^4+5x^3-3}$

ANSWERS

(1) $4x+3$ and 0 (2) $x+2$ and -12 (3) $x+1$ and -16 (4) $x+1$ and -1 (5) $3x+2$ and 0

(6) $3x+7$ and 6 (7) x^2-x-6 and 4 (8) $3x^2-4x+6$ and 0 (9) $5x^2-x+6$ and 0

(10) $5x^2+2x-3$ and 3 (11) x^2-x+2 and 0 (12) $2x^2+5x+2$ and 8

(13) $3x^2+7$ and 0 (14) $6x^2+18x+78$ and 313 (15) x^3-3x^2+3x-1 and 0

(16) $x^3-3x^2+8x-12$ and 15 (17) $x^2+5x+15$ and 41 (18) $2x^2+4x+9$ and 0

(19) $5x^2-4x+4$ and 0 (20) $2x^2+9x+18$ and 35 (21) $3x^2-12x+49$ and -200

(22) x^3+2x^2-2x+1 and 6 (23) x^2+4x-2 and 0 (24) x^2+2x+1 and $-3x-2$

(25) x^2-4 and 0 (26) x^3+x^2-3x-5 and $6x+11$

(27) $x+1$ and 0 (28) $3x^2-6x-1$ and $6x+3$ (29) $3x-1$ and $-30x+20$

(30) x^3-3 and 3 (31) $6x^2+x-12$ and 0 (32) $3x^2-x+2$ and 0

FACTORING

GREATEST COMMON FACTOR

$$ax + ab = a(x + b)$$

TRINOMIALS (with coefficient of $x^2 = 1$)

$$x^2 + (a+b)x + ab = (x+a)(x+b)$$

TRINOMIALS (with coefficient of $x^2 \neq 1$)

$$acx^2 + (ad+bc)x + bd = (ax+b)(cx+d)$$

SPECIAL FACTORIZATIONS

PERFECT SQUARE TRINOMIALS

$$x^2 + 2xy + y^2 = (x+y)^2$$

$$x^2 - 2xy + y^2 = (x-y)^2$$

DIFFERENCE OF SQUARES

$$x^2 - y^2 = (x+y)(x-y)$$

DIFFERENCE AND SUM OF CUBES

$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

$$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$

NOTE: A TRINOMIAL $ax^2 + bx + c$ can be factored over the integers only if $b^2 - 4ac = 0, 1, 4, 9, 16, \cdots$.

also, if $b^2 - 4ac = 0$, it is a perfect square trinomial.