Dawson College: **Functions and Trigonometry**: 201-009-50-S01: Fall 2008

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Test 1

This Test is graded out of 50. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* Simplify:

$$\frac{(-2xy^{-3}z^0)^{-3}}{(3xy^{-1}(xy)^{-1})^{-2}} = \frac{(-2)^{-3}x^{-3}y^{9}}{3^{-2}x^{-2}y^{2}(xy)^{2}}$$

$$= \frac{9x^2y^7}{-8x^3x^2y^2}$$

$$= \frac{-9y^5}{8x^3}$$

Question 2. *(3 marks)* Expand and simplify:

$$(x^2+y^2)[(x-y)(x+y)] = (x^2+y^2)(x^2-y^2)$$

$$= (x^4-y^4)$$

Question 3. *(3 marks)* Use long division to find the quotient and remainder:

$$\frac{x^4-6x^2+5x+4}{x-2}$$

$$\begin{array}{r|l}
 & x^3+2x^2-2x+1 \\
x-2 & x^4+0x^3-6x^2+5x+4 \\
 & -(x^4-2x^3) \\
 & \quad 2x^3-6x^2 \\
 & \quad -(2x^3-4x^2) \\
 & \qquad -2x^2+5x \\
 & \qquad -(-2x^2+4x) \\
 & \qquad\quad x+4 \\
 & \qquad\quad -(x-2) \\
 & \qquad\qquad 6
\end{array}$$

$$\therefore \frac{x^4-6x^2+5x+4}{x-2} = x^3+2x^2-2x+1 + \frac{6}{x-2}$$

Question 4. *(1 mark)* Factor:

$$16-9x^2 = 4^2 - 3^2x^2$$
$$= 4^2 - (3x)^2$$
$$= (4-3x)(4+3x)$$

Question 5. *(2 marks)* Factor:

$$4x^2-12x+9 = (2x-3)(2x-3)$$
$$= (2x-3)^2$$

Question 6. *(1 mark)* Factor:

$$x^2-13x+42 = (x-6)(x-7)$$

Question 7. *(2 mark)* Factor:

$$(x-2)^2+3(x-2) = (x-2)[(x-2)+3]$$
$$= (x-2)(x+1)$$

Question 8. *(3 marks)* Factor:

$$16x^5+48x^4+36x^3 = 4x^3(4x^2+12x+9)$$
$$= 4x^3(2x+3)(2x+3)$$
$$= 4x^3(2x+3)^2$$

Question 9. *(5 marks)* Simplify:

$$\frac{2x^2-x}{4x^2-1} \times \frac{4x^2+4x+1}{3x} \div \frac{4x^2-2x-2}{6x^2-6x} = \frac{x\cancel{(2x-1)}}{\cancel{(2x-1)}\cancel{(2x+1)}} \cdot \frac{\cancel{(2x+1)}\cancel{(2x+1)}}{\cancel{3x}} \cdot \frac{\cancel{6x}\cancel{(x-1)}}{\cancel{2}\cancel{(2x+1)}\cancel{(x-1)}}$$

$$= x$$

Question 10. *(5 marks)* Simplify:

$$\frac{x^2-11}{x^2+7x+6} - \frac{x}{x+6} + \frac{2}{x+1} = \frac{x^2-11}{(x+6)(x+1)} - \frac{x}{(x+6)} + \frac{2}{(x+1)}$$

LCD $= (x+6)(x+1)$

$$= \frac{x^2-11}{(x+6)(x+1)} - \frac{x(x+1)}{(x+6)(x+1)} + \frac{2(x+6)}{(x+6)(x+1)}$$

$$= \frac{x^2-11-x^2-x+2x+12}{(x+6)(x+1)}$$

$$= \frac{\cancel{x+1}}{(x+6)\cancel{(x+1)}} = \frac{1}{x+6}$$

Question 11. *(3 marks)* Simplify:

$$\sqrt{\frac{3}{x}}\left(1+\frac{3}{x}\right)\frac{2x^{3/2}}{\sqrt{3}} = \frac{\cancel{\sqrt{3}}}{\sqrt{x}}\left(1+\frac{3}{x}\right)\frac{2x^{3/2}}{\cancel{\sqrt{3}}}$$

$$= \left(\frac{x+3}{\cancel{x}}\right)\frac{2\cancel{\sqrt{x}}\,\cancel{x}}{\cancel{\sqrt{x}}}$$

$$= 2x+6$$

Question 12. *(2 marks)* Solve for x:

$$5(x-2) = 10 - (x+2)$$

$5x - 10 = 10 - x - 2$

$6x = 18$

$x = 3$

Question 13. *(2 marks each)* Rationalize the denominator:

a.

$$\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$$

b.

$$\frac{a}{1+\sqrt{a}}\left(\frac{1-\sqrt{a}}{1-\sqrt{a}}\right) = \frac{a(1-\sqrt{a})}{1-a}$$

Question 14. *(2 marks)* Solve for x by factoring:

$$4x^2 - 9 = 0$$

$0 = (2x-3)(2x+3)$

$2x - 3 = 0$ $\qquad$ $2x + 3 = 0$

$2x = 3$ $\qquad$ $2x = -3$

$x = \frac{3}{2}$ $\qquad$ $x = \frac{-3}{2}$

Question 15. *(2 marks)* Solve for x using the quadratic formula:

$$39 = 3(x^2+1)$$

$39 = 3x^2 + 3$

$0 = 3x^2 - 36$

$0 = x^2 - 12$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-0 \pm \sqrt{0^2 - 4(1)(-12)}}{2}$

$= \frac{\pm\sqrt{16 \cdot 3}}{2}$

$= \frac{\pm 4\sqrt{3}}{2}$

$= \pm 2\sqrt{3}$

Question 16. *(4 marks)* Solve for x using the quadratic formula(if needed):

$$(x^2-8)(4x^2-20x+25)=0$$

$x^2-8=0$

$x^2=8$

$x=\pm\sqrt{8}$

$4x^2-20x+25=0$

$(2x-5)^2=0$

$2x-5=0$

$2x=5$

$x=\frac{5}{2}$

$\therefore\ x=\pm\sqrt{8}\ ,\ 2$

Question 17. *(5 marks)* Solve for x:

$$x+\frac{14}{x-2}=\frac{7x}{x-2}+1$$

$LCD = x-2$

$$x(x-2)+\frac{14\cancel{(x-2)}}{\cancel{(x-2)}}=\frac{7x\cancel{(x-2)}}{\cancel{x-2}}+(x-2)$$

$x^2-2x+14=7x+x-2$

$0=x^2-10x+16$

$0=(x-2)(x-8)$

$x-2=0$

$x=2$

$x-8=0$

$x=8$

Check solution

	$x=2$	$x=8$
$x-2$	$=0$ not valid	$=6$ ok

$\therefore\ x=8$

Bonus

Let $ax^2+bx+c=0$ be a quadratic equation.

a. *(1 mark)* Define the discriminant Δ in terms of a, b, c.

b. *(1 mark)* Rewrite the quadratic formula using the discriminant Δ.

c. *(3 marks)* State the conditions on Δ for the number of solution of the quadratic equation, justify.

d. *(1 mark)* What is the condition on the discriminant, Δ, for the quadratic equation to be factorable over the integers, justify.

a) $\Delta = b^2 - 4ac$

b) $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

c) $\Delta > 0$, two solutions caused by the $\pm$ in front of the radical

$\Delta = 0$, one solution since $\pm$ has no effect.

$\Delta < 0$, no real solution since the radicand is negative

d) $\Delta =$ (perfect square) which will eliminate the radical hence a rational solution. The quadratic equation can be rewritten as

$$(2ax - (-b+\sqrt{\Delta}))(2ax - (-b-\sqrt{\Delta})) = 0$$