Dawson College: **Functions and Trigonometry**: 201-009-50-S01: Fall 2008

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Test 2

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.

a. *(4 marks)* Find the *distance* and the *midpoint* of the line segment between the points (2,3) and (4,3).

b. *(2 marks)* Find the equation of circle whose center is (2,3) and (4,3) is a point on the circle.

a) $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(4-2)^2+(3-3)^2}$

$= \sqrt{4}$

$= 2$

$(x_m, y_m) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{2+4}{2}, \frac{3+3}{2}\right) = (3,3)$

b) $(x-h)^2+(y-k)^2 = r^2$

$(x-2)^2+(y-3)^2 = 2^2$

$(x-2)^2+(y-3)^2 = 4$

Question 2. *(4 marks)* Use the x and y intercepts to graph the linear function.

$2x-3y=6$

x-int:

$y=0$

$2x=6$

$x=3$

$\therefore (3,0)$

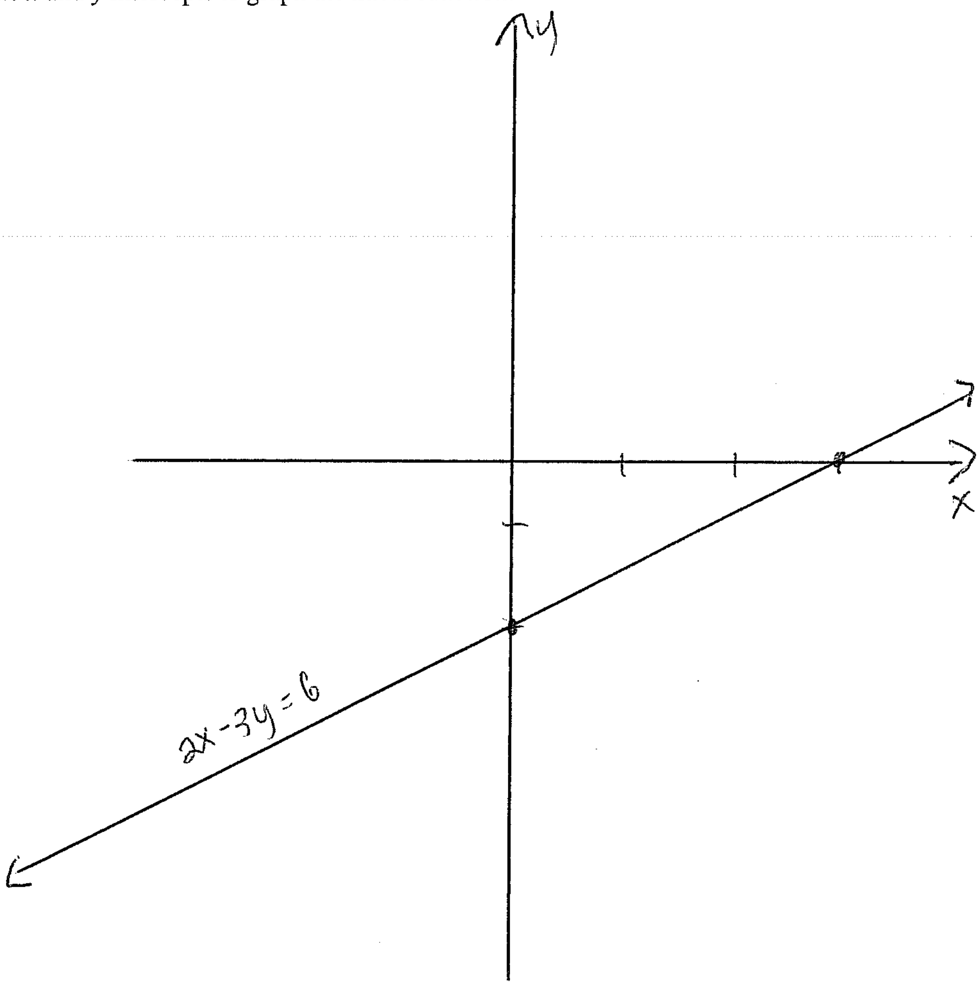
y-int:

$x=0$

$-3y=6$

$y=-2$

$\therefore (0,-2)$



Question 3. Let $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{x}{x^2+1}$.

a. *(4 marks)* Determine $\frac{f(x+h)-f(x)}{h}$ and simplify.

b. *(1 marks)* Determine the domain of $f(x)$.

c. *(2 marks)* Determine $(f \circ g)(x)$ and $(g \circ f)(x)$. *Do not simplify*

d. *(bonus 1 mark)* Determine the range of $f(x)$.

a) $$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h}$$

$LCD = (x+h+2)(x+2)$

$$= \frac{\frac{x+2}{(x+h+2)(x+2)} - \frac{x+h+2}{(x+2)(x+h+2)}}{h} = \frac{\frac{\cancel{x}+\cancel{2}-\cancel{x}-h-\cancel{2}}{(x+h+2)(x+2)}}{h}$$

$$= \frac{-\cancel{h}}{(x+h+2)(x+2)} \cdot \frac{1}{\cancel{h}}$$

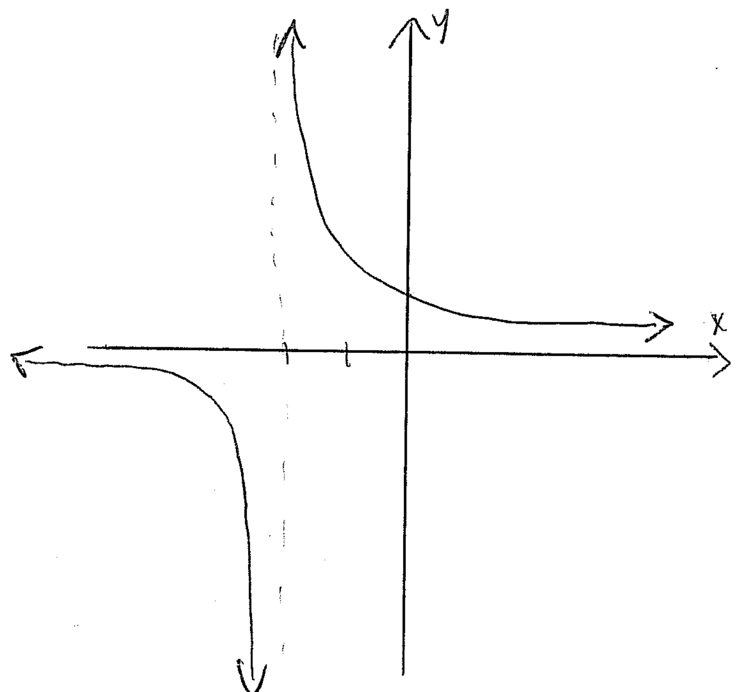
$$= \frac{-1}{(x+h+2)(x+2)}$$

b) Domain $f(x)$: all real numbers except -2.

c) $$(f \circ g)(x) = f(g(x)) = f\left(\frac{x}{x^2+1}\right) = \frac{1}{\frac{x}{x^2+1}+2}$$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x+2}\right) = \frac{\frac{1}{x+2}}{\left(\frac{1}{x+2}\right)^2 + 1}$$

d) Range: all real numbers except 0.



Question 4. Let $f(x) = 2x^2 - 8x + 6$ be a quadratic function.

a. *(2 marks)* Determine the vertex of $f(x)$ by completing the square.

b. *(1 mark)* Determine the orientation of the parabola and state whether the vertex is a minimum or maximum.

c. *(1 mark)* Determine the y-intercept.

d. *(1 mark)* Determine the x-intercept(s).

e. *(1 mark)* Sketch the graph of $f(x)$.

f. *(2 mark)* Determine the domain and range of $f(x)$.

a) $f(x) = 2(x^2-4x+3) = 2(x^2-4x+4-4+3)$

$= 2((x^2-4x+4)-1)$

$= 2((x-2)^2-1)$

$= 2(x-2)^2-2$

$\therefore$ vertex $(2,-2)$

b) $a=2>0$ $\cup$ $\therefore$ vertex a minimum

c) $(0,f(0))=(0,c)=(0,6)$

d) $0=f(x) \iff 0=2x^2-8x+6$

$0 = x^2-4x+3$

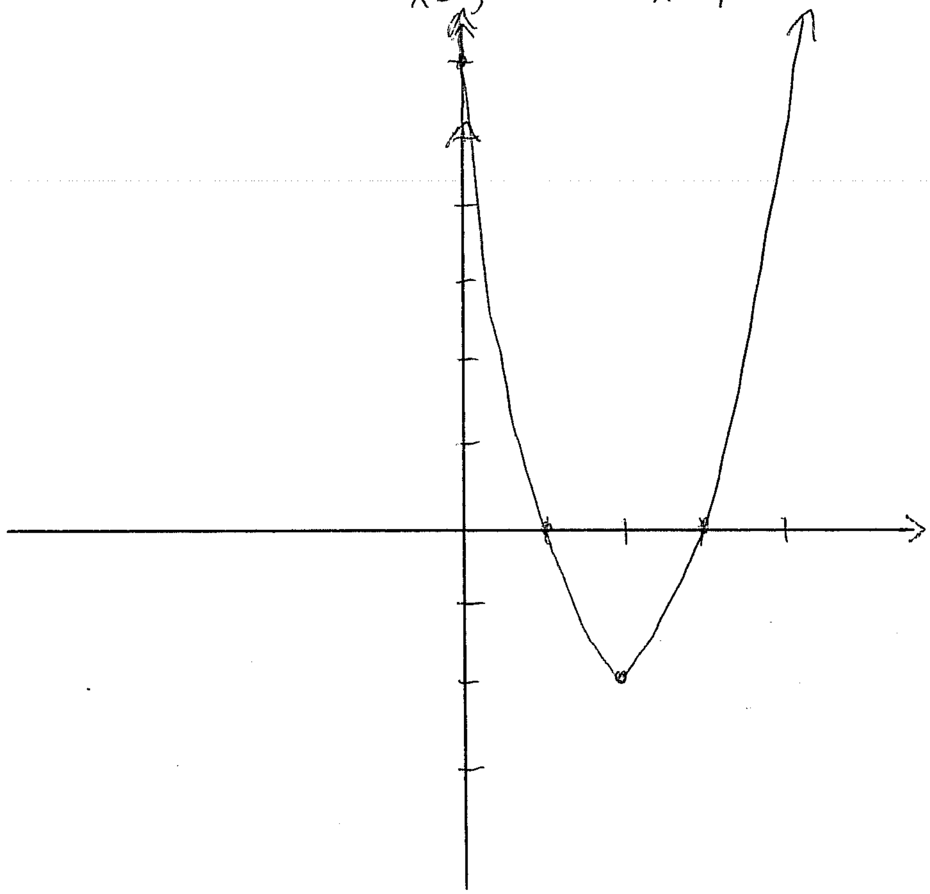
$0=(x-3)(x-1)$

$x-3=0$, $x=3$ $\qquad$ $x-1=0$, $x=1$

f) Domain: $\mathbb{R}$

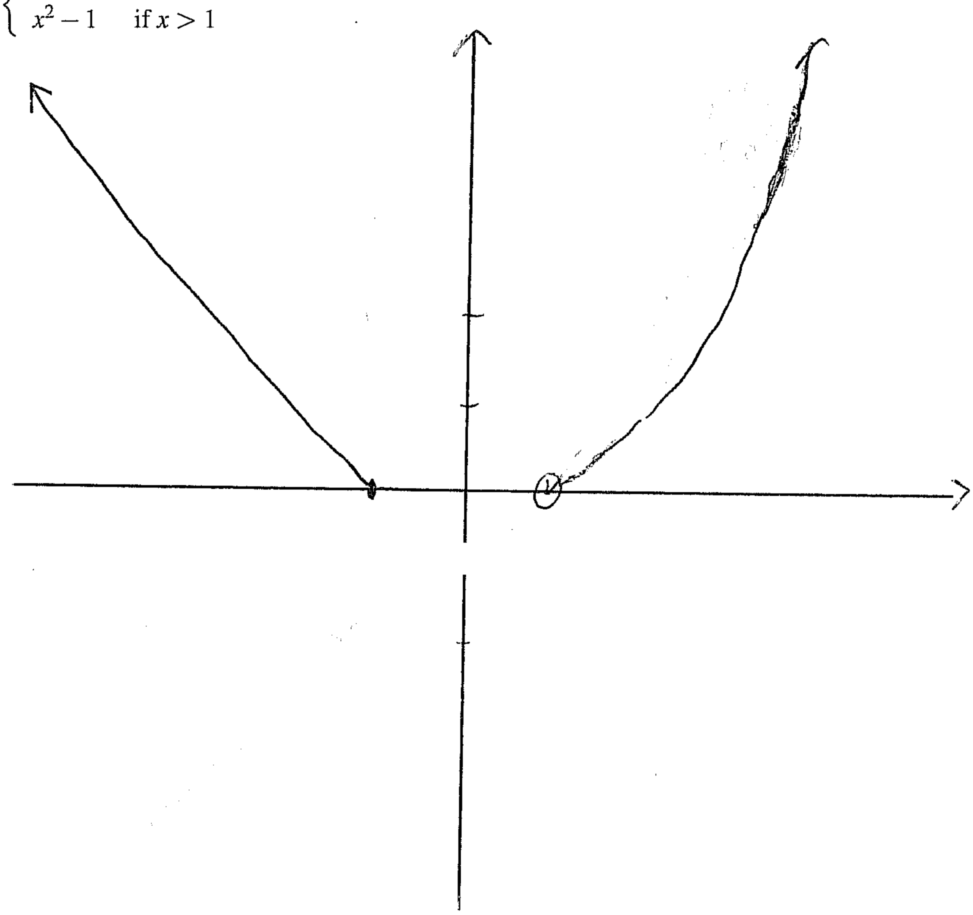
Range: $[-2,\infty)$

e)



Question 5. *(4 marks)* Sketch the graph defined by

$$f(x) = \begin{cases} -x-1 & \text{if } x \leq -1 \\ x^2-1 & \text{if } x > 1 \end{cases}$$



Question 6. *(4 marks)* Find the equation of the line that passes through the point $(1,2)$ and is perpendicular to the line $2x+5y=10$.

$2x+5y=10$

$5y=-2x+10$

$y=\frac{-2}{5}x+5$

$\therefore$ the slope of the $\perp$ is $\frac{5}{2}$

$\therefore y=mx+b$

$y=\frac{5}{2}x+b$

Solve for b

$2=\frac{5}{2}(1)+b$

$b=\frac{4}{2}-\frac{5}{2}=\frac{-1}{2}$

$\therefore y=\frac{5}{2}x-\frac{1}{2}$

Question 7. Let $f(x) = 2^{x-1} + 2$.

a. *(4 marks)* Find $f^{-1}(x)$.

b. *(4 marks)* Sketch the graph of $f(x)$ and $f^{-1}(x)$ on the same cartesian plane.

c. *(1 mark)* Determine the domain of $f^{-1}(x)$.

a)

$$f(x) = 2^{x-1} + 2$$

$$y = 2^{x-1} + 2$$

$$x = 2^{y-1} + 2$$ switch x and y in order to find the inverse

$$x - 2 = 2^{y-1}$$

$$\log_2(x-2) = \log_2(2^{y-1})$$

$$\log_2(x-2) = y - 1$$

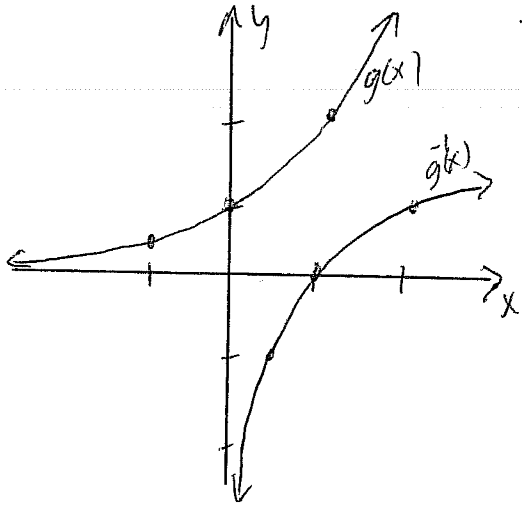
$$y = \log_2(x-2) + 1$$

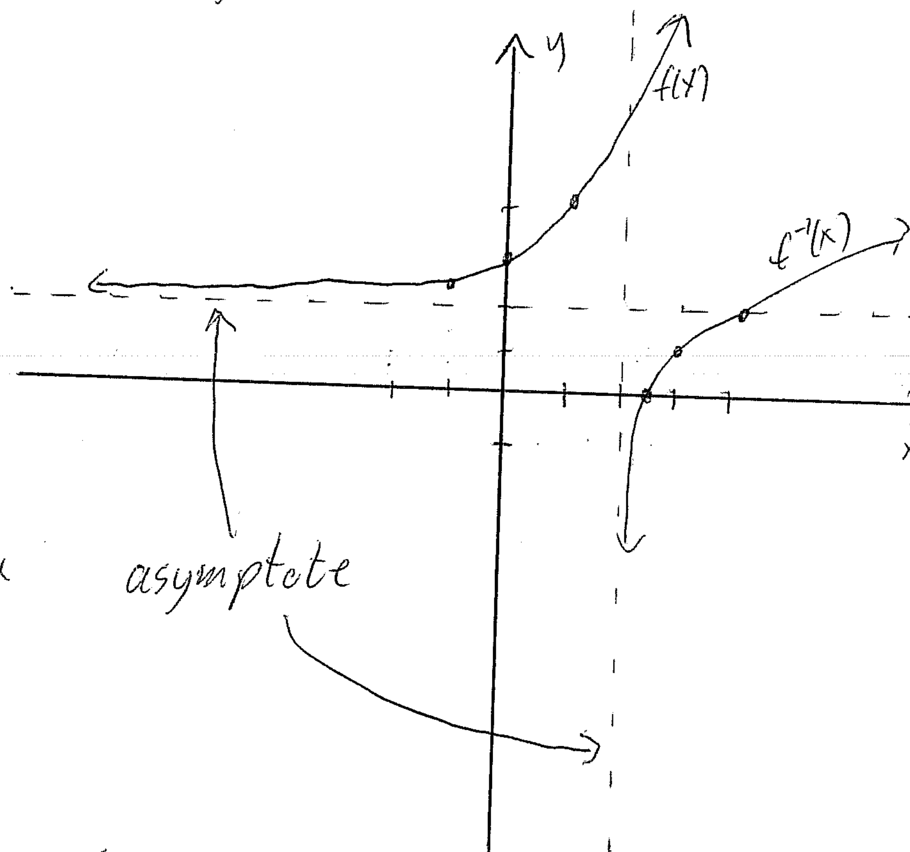
$\therefore f^{-1}(x) = \log_2 x$

b) First lets graph $g(x) = 2^x$ and $g^{-1}(x) = \log_2 x$

x	$g(x)$
-1	$\frac{1}{2}$
0	1
1	2

x	$g^{-1}(x)$
$\frac{1}{2}$	-1
1	0
2	1





c) Range of $f^{-1}(x)$: $(2, \infty)$

Question 8. Solve for x:

a. *(4 marks)* $\log_6(x+3) = 1 - \log_6(x+4)$

b. *(4 marks)* $27^{x-8} = \left(\frac{1}{3}\right)^{x+4}$

a) $\log_6(x+3) = 1 - \log_6(x+2))$

$1 = \log_6(x+3) + \log_6(x+4)$

$1 = \log_6[(x+3)(x+4)]$

$\log_6 6 = \log_6[(x+3)(x+4)]$

$6^{\log_6 6} = 6^{\log_6[(x+3)(x+4)]}$

$6 = (x+3)(x+4)$

$6 = x^2 + 7x + 12$

$0 = x^2 + 7x + 6$

$0 = (x+1)(x+6)$

$x+1 = 0$ $\quad$ $x+6=0$

$x = -1$ $\quad$ $x = -6$

verify solution

	$x=-1$	$x=-6$
$x+3$	2	-3
$x+4$	3	-2

$x=-6$ not a solution since a log would be evaluated at a negative

b) $27^{x-8} = \left(\frac{1}{3}\right)^{x+4}$

$3^{3(x-8)} = (3^{-1})^{x+4}$

$3^{3x-24} = 3^{-x-4}$

$\Rightarrow 3x - 24 = -x - 4$

$4x = 20$

$x = 5$

a) $1 - \log_{12} x = \log_{12}(x+1)$

$1 = \log_{12}(x+1) + \log_{12} x$

$1 = \log_{12} x(x+1)$

$12^1 = 12^{\log_{12} x(x+1)}$

$12 = x(x+1)$

$12 = x^2 + x$

$0 = x^2 + x - 12$

$0 = (x+4)(x-3)$

$x+4=0$ $\quad$ $x-3=0$

$x=-4$ $\quad$ $x=3$

verify solution

	$x=-4$	$x=3$
$x+4$	0	7
$x+3$	-1	6
	not valid	

$\therefore x = 3$

Bonus.

a. *(2 marks)* Prove that $f^{-1}(x) = \log_a(x)$ is the inverse of $f(x) = a^x$ by showing that $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$.

b. *(2 marks)* If $f(x) = 2x^2 - 1$ and $g(x) = x + 2$ then find all values of x such that $(g \circ f)(x) - [g(x)]^2 = 2$.

a) $y = \log_a x \iff x = a^y$

$$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f(\log_a x) = f(y) = a^y = x$$

$$(f^{-1} \circ f)(y) = f^{-1}(f(y)) = f^{-1}(a^y) = f^{-1}(x) = \log_a x = y$$

$\therefore$ $f^{-1}(x) = \log_a x$ is the inverse of $f(x)$.

b) $(g \circ f)(x) - [g(x)]^2 = 2$

$$2 = g(f(x)) - [x+2]^2$$
$$2 = g(2x^2-1) - (x^2+4x+4)$$
$$2 = 2x^2 - 1 + 2 - x^2 - 4x - 4$$
$$0 = x^2 - 4x - 5$$
$$0 = (x+1)(x-5)$$

$x + 1 = 0$, $x = -1$ $\qquad$ $x - 5 = 0$, $x = 5$

$\therefore$ $x = -1$ and $x = 5$