D awson College: Functions and Trigonometry: 201-009-50-S01: Fall 2008

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Test 3

This test is graded out of 46 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Identities:

$$\begin{aligned}\sin 2\theta &= 2\sin\theta\cos\theta\\ \cos 2\theta &= \cos^2\theta - \sin^2\theta\\ \cos 2\theta &= 1 - 2\sin\theta\\ \cos 2\theta &= 2\cos^2\theta - 1\end{aligned}$$

Question 1.

a. *(2 marks)* What angle θ ($0° \le \theta < 360°$) is co-terminal to $2045°$.

b. *(2 marks)* Consider an angle θ in standard position. Then find the quadrant that its terminal edge lies in, if $\sec\theta < 0$ and $\cot\theta < 0$.

c. *(4 marks)* Find the values of the other trigonometric functions, if $\csc\theta = -5$ and $\sec\theta < 0$.

a) $\theta_2 = \theta_1 + k360$ $\quad k = \left\lfloor \frac{2045}{360} \right\rfloor = 5$

$2045 = \theta_1 + 5 \cdot 360$

$\theta_1 = 245°$

b)

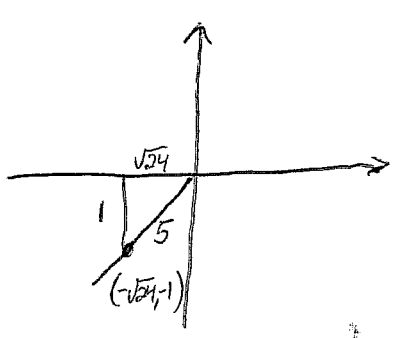
S $\sec\theta<0$ $\cot\theta<0$	A
$\sec\theta<0$	$\cot\theta<0$
T	C

∴ 2nd quadrant

c) $\csc\theta < 0$ and $\sec\theta < 0$

S $\sec\theta<0$	A
$\csc\theta<0$ $\sec\theta<0$	$\csc\theta<0$
T	C

∴ third quadrant



$\csc\theta = \frac{-5}{1} = \frac{\text{hyp.}}{\text{opp.}} \Rightarrow \sin\theta = \frac{-1}{5}$

$r = \sqrt{x^2+y^2}$

$5 = \sqrt{x^2+1^2}$

$25 = x^2+1$

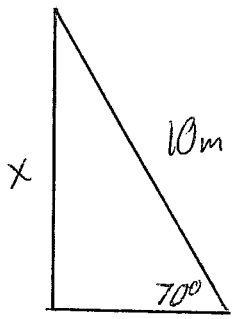
$24 = x^2$

$\sqrt{24} = x$

$\cos\theta = \frac{-\sqrt{24}}{5} \Rightarrow \sec\theta = \frac{-5}{\sqrt{24}}$

$\tan\theta = \frac{1}{\sqrt{24}} \Rightarrow \cot\theta = \sqrt{24}$

Question 2. *(4 marks)* A ladder leaning against a wall makes an angle of 70° with the ground. If the ladder is 10 meters long, how high up the wall does it reach?



$$\sin 70^\circ = \frac{x}{10}$$

$$10 \sin 70^\circ = x$$

$$x = 9.4\text{m}$$

Question 3.

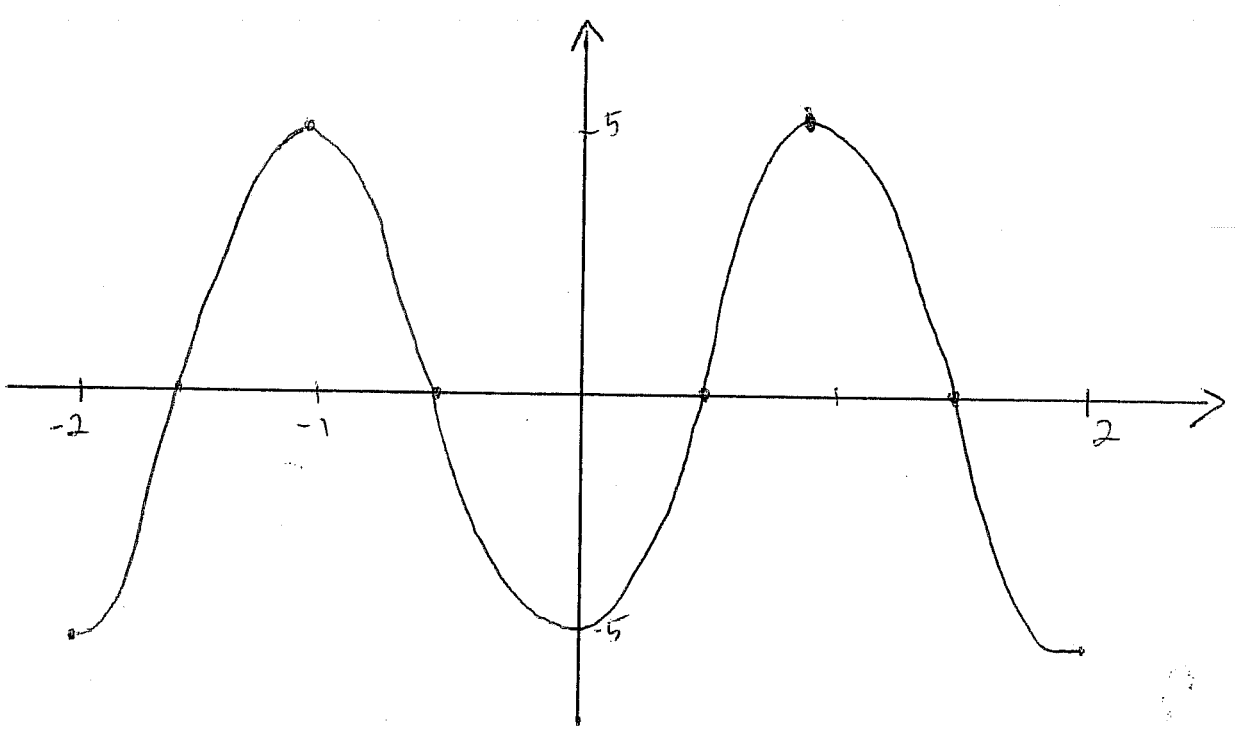
a. *(4 marks)* Sketch the graph of $y = -5\cos(\pi x)$ on the inteval $[-2,2]$.

b. *(2 marks)* State the amplitude and period of the above trigonometric function.

amplitude $= |-5| = 5$

period $= \frac{2\pi}{\pi} = 2$

a)



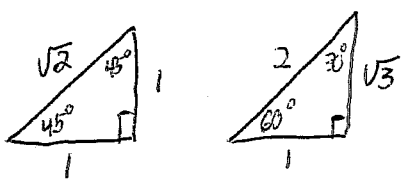
Question 4. Find the exact value.

a. *(4 marks)* Draw the two "special triangle" which help identify the special angles. Label the angles of the triangles and the lengths of the sides.

a. *(4 marks)* $\csc 945°$

b. *(4 marks)* $\tan \frac{5\pi}{3}$

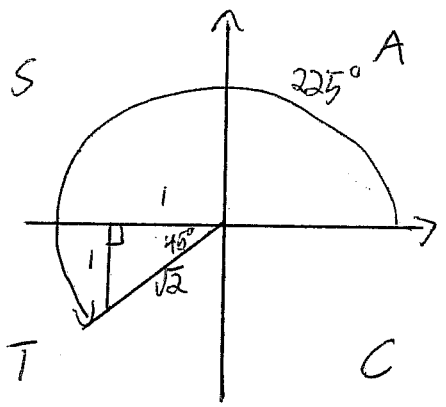
a)



a) $\csc 945° = \csc 225°$ $\quad \theta_R = 45°$

$= -\frac{hyp}{opp}$

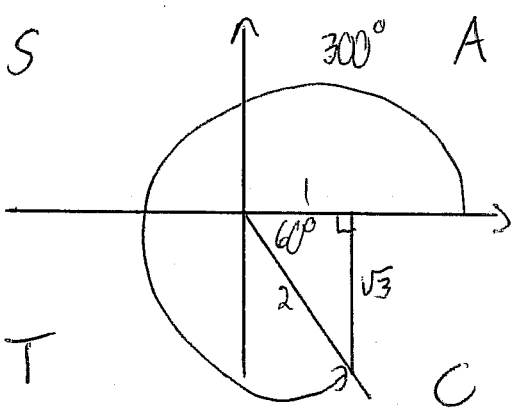
$= -\sqrt{2}$



b) $\tan \frac{5\pi}{3} = \tan 300°$ $\quad \theta_R = 60°$

$= -\frac{opp}{adj}$

$= -\sqrt{3}$



Question 5. Prove the following identities.

a. *(4 marks)*

$$\frac{\sin\theta}{\csc\theta}+\frac{\cos\theta}{\sec\theta}=1$$

b. *(4 marks)*

$$\frac{2\cot\theta}{\csc^2\theta}=\sin 2\theta$$

a) $\dfrac{\sin\theta}{\frac{1}{\sin\theta}} + \dfrac{\cos\theta}{\frac{1}{\cos\theta}} = 1$

$\sin^2\theta + \cos^2\theta = 1$

$1 = 1$

b) $\dfrac{2\cot\theta}{\frac{1}{\sin^2\theta}} = \sin 2\theta$

$\dfrac{\frac{2\cos\theta}{\sin\theta}}{\frac{1}{\sin^2\theta}} = \sin 2\theta$

$\dfrac{2\cos\theta \sin^2\theta}{\sin\theta} = \sin 2\theta$

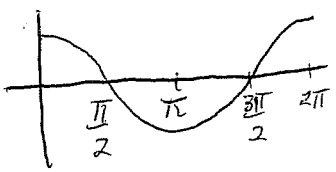
$2\cos\theta \sin\theta = 2\cos\theta \sin\theta$

Question 6.

, *(4 marks)* Solve for θ, giving the exact solution, $0 \leq \theta < 2\pi$

$\cos\theta(4\sin^2\theta - 1) = 0$

$\cos\theta = 0$

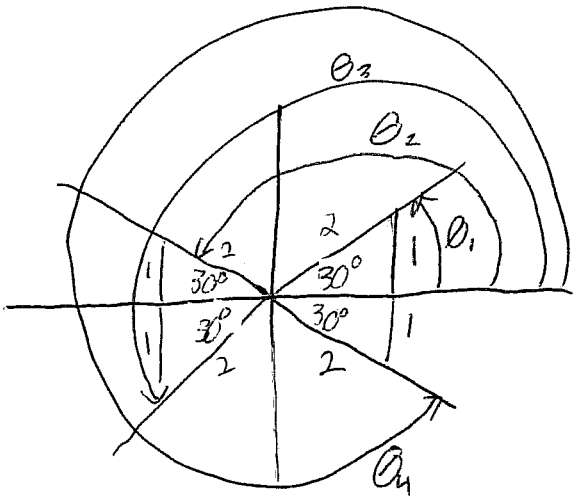


$\therefore \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$4\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\frac{1}{2}$



note $30° = \frac{\pi}{6}$

$\theta_1 = \frac{\pi}{6}$

$\theta_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\theta_3 = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\theta_4 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\therefore \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

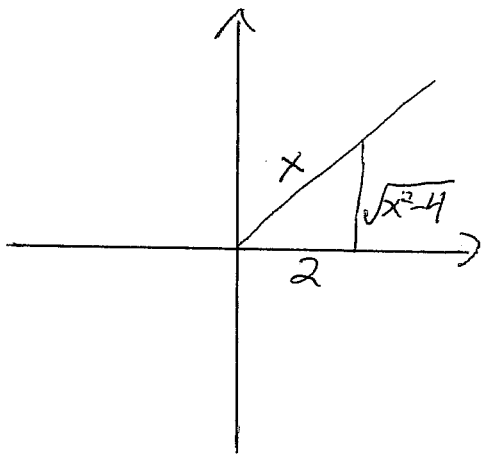
Question 7. *(4 marks)* Find the expression of:

$$\tan\left[\arccos\left(\frac{2}{x}\right)\right]$$

$\cos\theta = \frac{adj}{hyp}$

$= \frac{opp}{adj}$

$= \frac{\sqrt{x^2-4}}{2}$



Bonus.

a. *(2 marks)* State the domain and range of $f(x) = \cot x$.

b. *(2 marks)* Sketch the graph of $f(x) = \cot x$ on the interval $[0, \pi]$.

c. *(2 marks)* Sketch the graph of $f^{-1}(x)$ if $f(x) = \cot x$ is restricted on $[0, \pi]$.

a) Range $f(x)$: $\mathbb{R}$

Domain $f(x)$: all angles except $\{\pm 0, \pm\pi, \pm 2\pi, \ldots\}$

$\cot x = \frac{\cos x}{\sin x}$