

**TEST 3 (201-009-DW)**  
**Functions & Trigonometry**

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This test is marked out of 60 marks.  
 Scientific calculator is permitted.  
 SHOW ALL YOUR WORK.

**Question 1 (4 marks)**

Find the inverse of the function  $f(x) = \frac{x+2}{x}$

$$y = \frac{x+2}{x}$$

Reverse  $x$  &  $y$

$$x = \frac{y+2}{y} \implies xy = y+2$$

$$xy - y = 2$$

$$y(x-1) = 2$$

$$y = \frac{2}{x-1} \implies$$

$$f^{-1}(x) = \frac{2}{x-1}$$

**Question 2 (3 marks)**

Solve the following equation for  $x$

$$\ln(x+1) + \ln x = \ln 2$$

$$\ln((x+1)x) = \ln 2$$

$$\ln(x^2+x) = \ln 2$$

$$x^2+x = 2$$

$$x^2+x-2 = 0$$

USE QUADRATIC

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2}$$

$$= \frac{-1 \pm \sqrt{9}}{2} = \frac{-1 \pm 3}{2} \implies$$

$$x = -2 \text{ or } x = 1$$

only  $x=1$  is a solution

**Question 3 (4 marks)**

Solve the following equation for x

$$\log_3(2x+1) + \log_3(x-2) = 2$$

$$\log_3((2x+1)(x-2)) = 2$$

$$\log_3(2x^2 - 3x - 2) = 2$$

$$3 \log_3(2x^2 - 3x - 2) = 3^2$$

$$2x^2 - 3x - 2 = 9$$

$$2x^2 - 3x - 11 = 0$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-11)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{97}}{4} \implies x = 3.21 \text{ or } x = -1.71$$

But only  $x = 3.21$  is a solution

**Question 4 (4 marks)**

Solve the following equation for x

$$2(5^{x+1}) = 3^{1-x}$$

$$\ln(2(5^{x+1})) = \ln(3^{1-x})$$

$$\ln 2 + \ln 5^{x+1} = \ln 3^{1-x}$$

$$\ln 2 + (x+1)\ln 5 = (1-x)\ln 3$$

$$\ln 2 + \ln 5x + \ln 5 = \ln 3 - x\ln 3$$

$$x\ln 5 + x\ln 3 = \ln 3 - \ln 5 - \ln 2$$

$$x(\ln 5 + \ln 3) = \ln 3 - \ln 5 - \ln 2$$

$$x = \frac{\ln 3 - \ln 5 - \ln 2}{\ln 5 + \ln 3}$$

OR

$$x = -0.44$$

**Question 5 (5 marks)**

Solve the following equation for x

$$7^{2x} - 4(7^x) + 3 = 0$$

$$(7^x)^2 - 4(7^x) + 3 = 0$$

$$\text{Let } y = 7^x \Rightarrow y^2 - 4y + 3 = 0$$

$$(y-3)(y-1) = 0$$

$$y = 3 \text{ or } y = 1$$

$$\Rightarrow 3 = 7^x$$

$$\text{OR } 1 = 7^x$$

$$\ln 3 = x \ln 7$$

$$\ln 1 = x \ln 7$$

$$\boxed{x = \frac{\ln 3}{\ln 7}}$$

$$\boxed{x = 0}$$

**Question 6 (3 marks)**

Convert the following values into degrees

(a)  $\frac{5\pi}{4}$

(b)  $12\pi$

(c)  $\frac{11\pi}{7}$

$$(a) \frac{5\pi}{4} \cdot \frac{180}{\pi} = \boxed{225^\circ}$$

$$(b) 12\pi = 12\pi \left(\frac{180}{\pi}\right) = \boxed{2160^\circ}$$

$$(c) \frac{11\pi}{7} \cdot \frac{180}{\pi} = \boxed{282.86^\circ}$$

**Question 6 (6 marks)**

- (a) Find the inverse of the function  $f(x) = \log_2(x+1)$   
 (b) Sketch the function  $f$  as well as its inverse  $f^{-1}$  on the same graph  
 (c) State the domain and range of the functions  $f$  and  $f^{-1}$

(a)  $y = \log_2(x+1)$

inverse

$$x = \log_2(y+1)$$

$$2^x = y+1$$

$$y = 2^x - 1$$

$$f^{-1}(x) = 2^x - 1$$

(b)  $y = \log_2(x+1)$

$$2^y = x+1$$

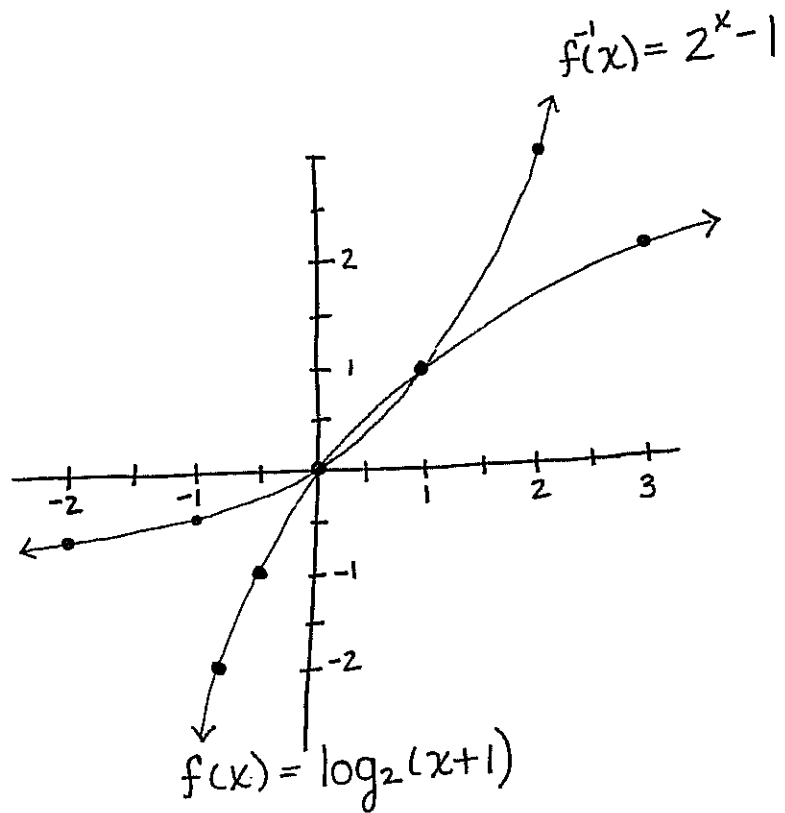
$$x = 2^y - 1$$

x	y
-3/4	-2
-1/2	-1
0	0
1	1
3	2

inverse

$$y = 2^x - 1$$

x	y
-2	-3/4
-1	-1/2
0	0
1	1
2	3



(c)  $f$ : DOMAIN  $x > -1$   $(-1, \infty)$   
 RANGE  $(-\infty, \infty)$

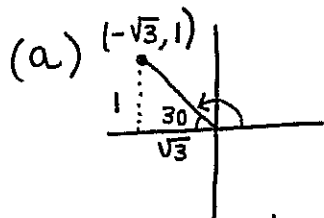
$f^{-1}$ : DOMAIN  $(-\infty, \infty)$   
 RANGE  $(-1, \infty)$

**Question 7 (6 marks)**Find the exact value (no decimals) of the following

(a)  $\sec(150^\circ)$

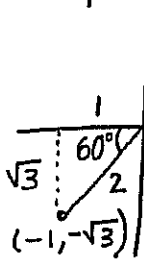
(b)  $\cot(240^\circ)$

(c)  $\sin(-855^\circ)$



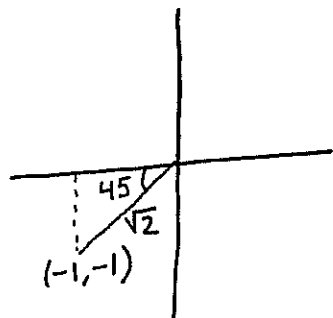
$$\sec(150^\circ) = \frac{r}{x} = \frac{2}{-\sqrt{3}}$$

(b)



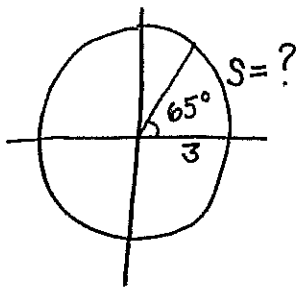
$$\cot(240^\circ) = \frac{x}{y} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$$

(c)



$-855^\circ$  is co-terminal to  $225^\circ$

$$\sin(-855^\circ) = \frac{y}{r} = \frac{-1}{\sqrt{2}}$$

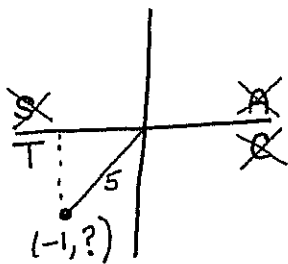
**Question 8 (3 marks)**Find the arc length of a circle of radius 3cm subtended by a central angle of  $65^\circ$ 

$$65^\circ = 65 \left( \frac{\pi}{180} \right) = \frac{13\pi}{36}$$

$$\theta_{\text{rad}} = \frac{S}{r}$$

$$\frac{13\pi}{36} = \frac{S}{3}$$

$$S = \frac{13\pi}{12}$$

**Question 9 (5 marks)**Find the values of the five other trigonometric functions if  $\sec\theta = -5$  and  $\csc\theta < 0$ 

$$\sec\theta = \frac{r}{x} = \underline{-5}$$

$$r = 5$$

$$x = -1$$

By PYTHAGORUS  $r^2 = x^2 + y^2$

$$25 = 1 + y^2$$

$$y = \pm\sqrt{24}$$

$$y = -\sqrt{24} \quad (\text{BECAUSE OF QUADRANT})$$

$$\sin\theta = \frac{-\sqrt{24}}{5}$$

$$\cos\theta = \frac{-1}{5}$$

$$\tan\theta = \frac{-\sqrt{24}}{-1} = \sqrt{24}$$

$$\csc\theta = -\frac{5}{\sqrt{24}}$$

$$\sec\theta = -5$$

$$\cot\theta = \frac{1}{\sqrt{24}}$$

**Question 10 (5 marks)**Find the values of the five other trigonometric functions if  $\cot\theta = 0$  and  $\sin\theta = -1$ 

$$\cot\theta = 0$$

$$\frac{x}{y} = 0 \quad \underline{x=0}$$

$$\sin\theta = -1 = \frac{y}{r} \rightarrow \underline{\underline{y=-1}}$$

$$\underline{\underline{r=1}}$$

$$\sin\theta = -1$$

$$\csc\theta = -1$$

$$\cos\theta = 0$$

$$\sec\theta \text{ does NOT EXIST}$$

$$\tan\theta \text{ does not EXIST}$$

$$\cot\theta = 0$$

**Question 11** (6 marks)

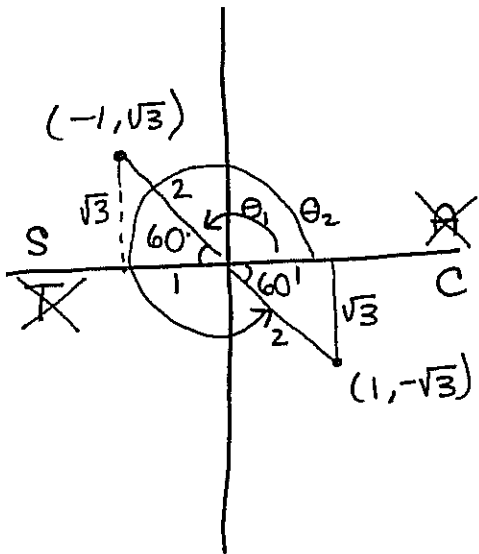
Solve for  $\theta$  giving exact solutions (in radians).

Your solutions should be in the range  $0 \leq \theta < 2\pi$ .

(a)  $\tan\theta + \sqrt{3} = 0$

$$\tan\theta = -\sqrt{3}$$

$$\frac{y}{x} = -\sqrt{3}$$



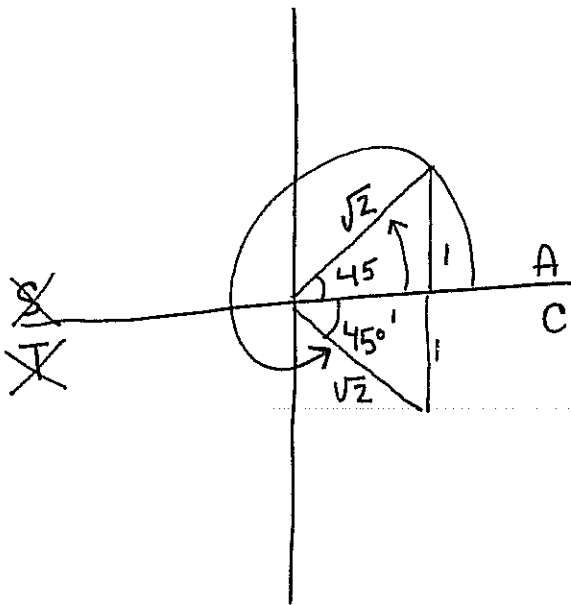
$$\theta_1 = 120^\circ = \boxed{\frac{2\pi}{3}}$$

$$\theta_2 = 300^\circ = \boxed{\frac{5\pi}{3}}$$

(b)  $\sec\theta = \sqrt{2}$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{2}}{1}$$

$$\begin{aligned} r &= \sqrt{2} \\ x &= 1 \end{aligned}$$



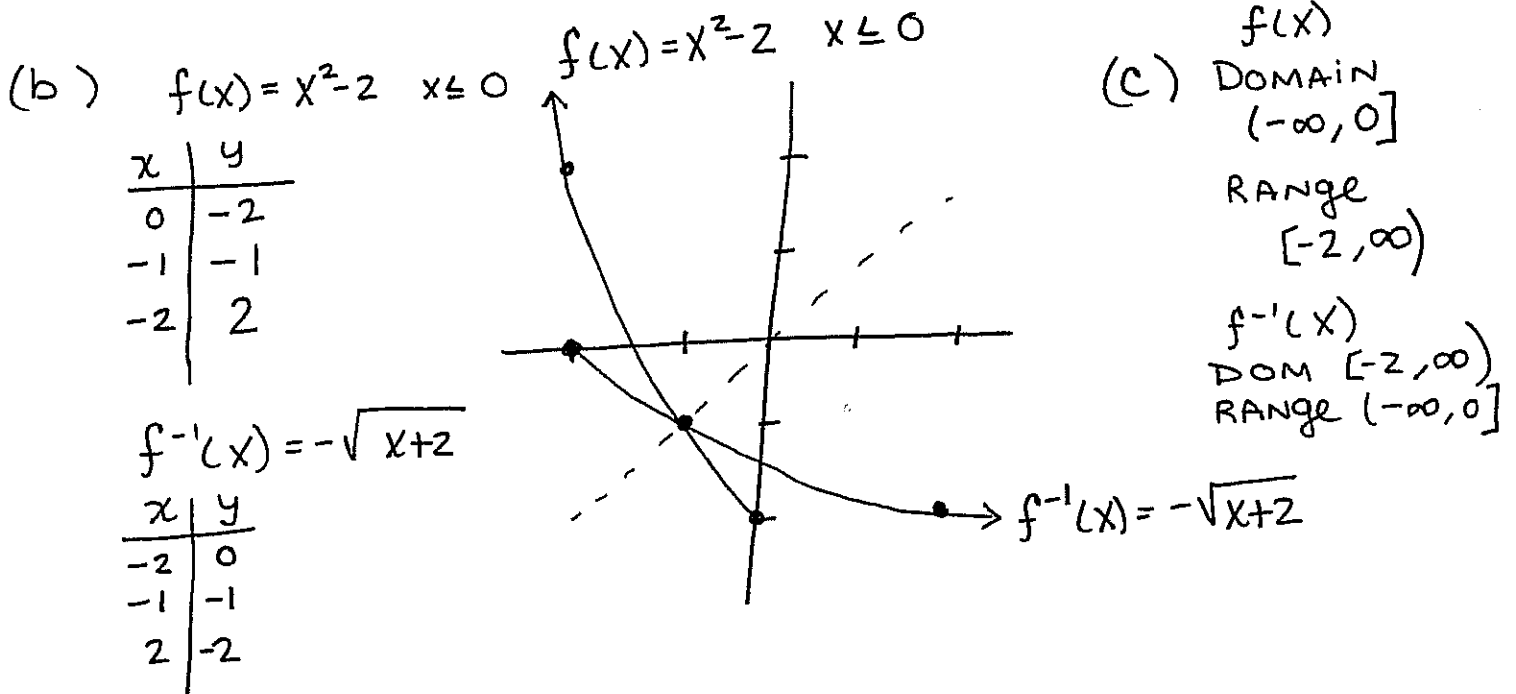
$$\theta_1 = 45^\circ = \boxed{\frac{\pi}{4}}$$

$$\theta_2 = 315^\circ = \boxed{\frac{7\pi}{4}}$$

**Question 12 (6 marks)**

- (a) Find the inverse of the function  $f(x) = x^2 - 2, x \leq 0$   
 (b) Sketch the function  $f$  as well as its inverse  $f^{-1}$  on the same graph  
 (c) State the domain and range of the functions  $f$  and  $f^{-1}$

(a)  $x = y^2 - 2$   
 $y^2 = x + 2$   
 $y = -\sqrt{x+2}$  (because  $x \leq 0$  is a restriction for  $f$ )  
 $f^{-1}(x) = -\sqrt{x+2}$



**BONUS (3 marks)**

Show all the steps required to evaluate the following expressions without a calculator:

$$(\log_2 3)(\log_3 32) = (\cancel{\log_2 3}) \cdot \frac{(\log_2 32)}{(\cancel{\log_2 3})} \quad (\text{by base conversion})$$

$$= \log_2(2^5)$$

$$= 5$$