

**TEST 1 (201-009-DW)**  
**Functions & Trigonometry**

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This test is marked out of marks 60 marks.

Scientific calculator is permitted.

SHOW ALL YOUR WORK.

**Question 1 (9 marks)**

Simplify the following expressions so that only positive exponents remain

(a) (3 marks)  $(-2a^{-2}b^{-3})(2a^3b^{-4})(2^{-3}ab^{-2})$

$$= \left(\frac{-2}{a^2b^3}\right)\left(\frac{2a^3}{b^4}\right)\left(\frac{a}{2^3b^2}\right)$$

$$= \frac{-4a^4}{8a^2b^9} = \boxed{\frac{-a^2}{2b^9}}$$

(b) (3 marks)  $\frac{(-3a^2b^{-2}c)^2}{(3a^{-3}b^4c)^{-1}}$

$$= \frac{(-3)^2 a^4 b^{-4} c^2}{3^{-1} a^3 b^{-4} c^{-1}} = \frac{9a^4 c^2 3^1 b^4 c^1}{a^3 b^4}$$

$$= \boxed{27ac^3}$$

(c) (3 marks)  $\frac{(-3x^{-4}y^3)^{-2}(9xy)^2}{(x^2y)^0(x^2y^{-3})^3} = \frac{(-3)^{-2} x^8 y^{-6} 9^2 x^2 y^2}{x^6 y^{-9}}$

$$= \frac{9 \cdot 9^2 x^{10} y^{-4}}{x^6 y^{-9}}$$

$$= \boxed{9^3 x^4 y^5}$$

**Question 2 (6 marks)**

Simplify the following radical expressions.

(a) (1.5 marks)  $3\sqrt{20} - 4\sqrt{45} + \sqrt{80}$

$$\begin{aligned} &= 3\sqrt{4 \cdot 5} - 4\sqrt{9 \cdot 5} + \sqrt{16 \cdot 5} \\ &= 3\sqrt{4} \cdot \sqrt{5} - 4\sqrt{9} \cdot \sqrt{5} + \sqrt{16} \cdot \sqrt{5} \\ &= 3 \cdot 2 \cdot \sqrt{5} - 4 \cdot 3 \cdot \sqrt{5} + 4 \cdot \sqrt{5} \\ &= 6\sqrt{5} - 12\sqrt{5} + 4\sqrt{5} \\ &= -2\sqrt{5} \end{aligned}$$

(b) (1.5 marks)  $(2\sqrt{6})(3\sqrt{3})$

$$\begin{aligned} &= 6\sqrt{6} \cdot \sqrt{3} \\ &= 6\sqrt{6 \cdot 3} \\ &= 6\sqrt{18} \\ &= 6\sqrt{9 \cdot 2} = 6\sqrt{9}\sqrt{2} \\ &= 6 \cdot 3\sqrt{2} = \boxed{18\sqrt{2}} \end{aligned}$$

(c) (1.5 marks)  $(-8)^{5/3} + 8^{2/3} - 4^{5/2}$

$$\begin{aligned} &= \left((-8)^{1/3}\right)^5 + \left(8^{1/3}\right)^2 - \left(4^{1/2}\right)^5 \\ &= \left(\sqrt[3]{-8}\right)^5 + \left(\sqrt[3]{8}\right)^2 - \left(\sqrt{4}\right)^5 \\ &= (-2)^5 + (2)^2 - 2^5 \\ &= -32 + 4 - 32 = \boxed{-60} \end{aligned}$$

(d) (1.5 marks)  $(4^5)^{-3/5} - [(-8)^6]^{-1/3}$

$$\begin{aligned} &= 4^{5 \cdot (-3/5)} - (-8)^{6 \cdot (-1/3)} \\ &= 4^{-3} - (-8)^{-2} \\ &= \frac{1}{4^3} - \frac{1}{(-8)^2} = \frac{1}{64} - \frac{1}{64} = \boxed{0} \end{aligned}$$

**Question 3** (14 marks)

Perform the indicated operations and simplify your answer.

(a) (2 marks)  $-(8 - 4(x + 5) + x)$

$$\begin{aligned} &= -(8 - 4x - 20 + x) \\ &= -(8 - 3x - 20) \\ &= -(-12 - 3x) \\ &= 12 + 3x \end{aligned}$$

(b) (2 marks)  $x^3 - [x^2 - (x^3 + x^2)] - [x^3 + (1 - x^2)]$

$$\begin{aligned} &= x^3 - [x^2 - x^3 - x^2] - [x^3 + 1 - x^2] \\ &= x^3 - [-x^3] - [x^3 + 1 - x^2] \\ &= x^3 + x^3 - x^3 - 1 + x^2 \\ &= \boxed{x^3 + x^2 - 1} \end{aligned}$$

(c) (3 marks)  $-x^3(2x + 1)^2$

$$\begin{aligned} &= -x^3(2x + 1)(2x + 1) \\ &= -x^3(4x^2 + 2x + 2x + 1) \\ &= -x^3(4x^2 + 4x + 1) \\ &= \boxed{-4x^5 - 4x^4 - x^3} \end{aligned}$$

(d) (3 marks)  $(2y - 2)(y + 1) - (3y + 5) + 2$

$$\begin{aligned} &= 2y^2 + 2y - 2y - 2 - 3y - 5 + 2 \\ &= \boxed{2y^2 - 3y - 5} \end{aligned}$$

(e) (4 marks)  $(6x^4 - 2x + 2) \div (x - 2)$

(Express your answer in one of the two ways shown in class)

$$\begin{array}{r} 6x^3 + 12x^2 + 24x + 46 \\ x-2 \overline{) 6x^4 - 2x + 2} \\ \underline{-(6x^4 - 12x^3)} \phantom{+ 2} \\ 12x^3 - 2x + 2 \\ \underline{-(12x^3 - 24x^2)} \phantom{+ 2} \\ 24x^2 - 2x + 2 \\ \underline{-(24x^2 - 48x)} \phantom{+ 2} \\ 46x + 2 \\ \underline{-(46x - 92)} \\ 94 \end{array}$$

ANSWER  $\boxed{\frac{6x^4 - 2x + 2}{x - 2} = 6x^3 + 12x^2 + 24x + 46 + \frac{94}{x - 2}}$

Question 4 (5 marks)

Simplify completely.

$$\frac{2x^2 + 8x - 42}{x - 3} \div \frac{2x^2 + 14x}{x^2 + 5x}$$

$$= \frac{2(x^2 + 4x - 21)}{x - 3} \cdot \frac{x^2 + 5x}{2x^2 + 14x}$$

$$= \frac{\cancel{2}(x + 7)(x - 3)}{x - 3} \cdot \frac{\cancel{x}(x + 5)}{\cancel{2x}(x + 7)}$$

$$= \boxed{x + 5}$$

**Question 5** (5 marks)

Perform the indicated operations and simplify.

$$\begin{aligned} & \frac{4a+1}{a-8} - \frac{3a+2}{a+4} - \frac{49a+4}{a^2-4a-32} \\ &= \frac{4a+1}{a-8} - \frac{3a+2}{a+4} - \frac{49a+4}{(a-8)(a+4)} \\ &= \frac{(4a+1)(a+4)}{(a-8)(a+4)} - \frac{(3a+2)(a-8)}{(a+4)(a-8)} - \frac{49a+4}{(a-8)(a+4)} \\ &= \frac{(4a^2+a+16a+4) - (3a^2+2a-24a-16) - (49a+4)}{(a-8)(a+4)} \\ &= \frac{a^2-10a+16}{(a-8)(a+4)} = \frac{(a-8)(a-2)}{(a-8)(a+4)} = \boxed{\frac{a-2}{a+4}} \end{aligned}$$

**Question 6** (5 marks)

Simplify completely.

$$\begin{aligned} & \frac{x^2+3x+2}{x^2+5x+4} \cdot \frac{x^2+10x+24}{x^2+5x+6} \div \frac{2x^2+11x-6}{-2x^2-5x+3} \\ &= \frac{\cancel{(x+2)}\cancel{(x+1)}}{\cancel{(x+4)}\cancel{(x+1)}} \cdot \frac{\cancel{(x+6)}\cancel{(x+4)}}{\cancel{(x+3)}\cancel{(x+2)}} \cdot \frac{-\cancel{(2x-1)}\cancel{(x+3)}}{\cancel{(2x-1)}\cancel{(x+6)}} \\ &= \boxed{-1} \end{aligned}$$

FACTORING

$$\begin{aligned} & 2x^2+11x-6 \\ &= 2x^2+12x-x-6 \\ &= 2x(x+6)-1(x+6) \\ &= (2x-1)(x+6) \\ & -2x^2+5x+3 \\ &= -(2x^2+5x-3) \\ &= -(2x^2+6x-x-3) \\ &= -(2x(x+3)-1(x+3)) \\ &= -(2x-1)(x+3) \end{aligned}$$

**Question 7 (4 marks)**

Factor completely.

(a) (2 marks)  $7x^4 + 7x^3 - 140x^2$

$$= 7x^2(x^2 + x - 20)$$

$$= 7x^2(x+5)(x-4)$$

(b) (2 marks)  $25x^2 + 65x - 30$

$$= 5(5x^2 + 13x - 6)$$

$$= 5(5x^2 + 15x - 2x - 6)$$

$$= 5(5x(x+3) - 2(x+3))$$

$$= 5(5x-2)(x+3)$$

**Question 8 (4 marks)**

Perform the indicated operations and simplify.

$$\frac{x - \frac{9}{x}}{x+4 + \frac{3}{x}} = \frac{\frac{x^2 - 9}{x}}{\frac{(x+4)x + 3}{x}}$$

$$= \frac{\frac{x^2 - 9}{x}}{\frac{x^2 + 4x + 3}{x}} = \frac{x^2 - 9}{x} \cdot \frac{x}{x^2 + 4x + 3}$$

$$= \frac{(x+3)(x-3)}{(x+3)(x+1)}$$

$$= \boxed{\frac{x-3}{x+1}}$$

**Question 9** (4 marks)

Rationalize the denominator and simplify.

$$\begin{aligned} & \frac{7+3\sqrt{3}}{2-2\sqrt{3}} \cdot \frac{(2+2\sqrt{3})}{2+2\sqrt{3}} \\ &= \frac{7 \cdot 2 + 6\sqrt{3} + 14\sqrt{3} + (3\sqrt{3})(2\sqrt{3})}{4 - 4(3)} \\ &= \frac{14 + 20\sqrt{3} + 18}{-8} \\ &= \frac{32 + 20\sqrt{3}}{-8} = \frac{2(16 + 10\sqrt{3})}{-8} \\ &= \left( \frac{16 + 10\sqrt{3}}{-4} \right) = \boxed{-4 - \frac{5\sqrt{3}}{2}} \end{aligned}$$

**Question 10** (4 marks)

Perform the multiplication and simplify.

$$\begin{aligned} & (3\sqrt{5} - 4\sqrt{2})(2\sqrt{5} + 3\sqrt{2}) \\ &= 6 \cdot 5 - 8\sqrt{2}\sqrt{5} + 9\sqrt{2}\sqrt{5} - 12 \cdot 2 \\ &= 30 + \sqrt{2}\sqrt{5} - 24 \\ &= 30 + \sqrt{10} - 24 \\ &= \boxed{6 + \sqrt{10}} \end{aligned}$$