Final Examination
Functions & Trigonometry
(201-009-50 / 912-015-94)
13 December 2005

1. **(3 points)** Perform the division: $\dfrac{2x^3 - x^2 + 3x + 2}{2x + 1}$.

2. **(3 points)** Factor completely: $8x^3 - 27$.

3. **(3 points)** Simplify: $\dfrac{x^2 - 3x - 10}{x^2 - 5x} \div \dfrac{x^2 - 4}{x^3 + x^2 - 6x}$.

4. **(3 points)** Simplify: $\dfrac{1 + \frac{b}{a}}{\frac{a^2}{b} - b}$.

5. **(3 points)** Rationalize the denominator and simplify: $\dfrac{\sqrt{12}}{\sqrt{3} + 1}$.

6. **(3 points)** Solve for x: $\dfrac{x}{x+2} + \dfrac{x}{2-x} = \dfrac{x+20}{x^2 - 4}$.

7. **(3 points)** Solve for x: $5x^2 = 12 - 4x$.

8. **(2 points)** State the equation of the circle with center $(-4, 5)$ and radius 6.

9. **(3 points)** Find the equation of the line through $(4, 11)$ that is perpendicular to the line $2x + 3y = 7$.

10. **(12 points)** Consider: $f(x)=\frac{4}{x+3}$ and $g(x)=2x^2-3$.

(a) State the domain of $f(x)$.

(b) Find $\dfrac{g(x+h)-g(x)}{h}$ and simplify.

(c) Find $f^{-1}(x)$.

(d) Find $(fog)(-2)$, that is $f\left[g(-2)\right]$.

11. **(9 points)** Given points A (4, –1) and B (0, 3), find the:

(a) Slope of the line passing through A and B.

(b) Midpoint of the line segment AB.

(c) Distance from A to B.

12. **(6 points)** Sketch the graph of $f(x)=-5x^2-10x$ indicating the coordinates of the vertex and the intercepts, and state the range.

13. **(3 points)** Graph $y=2^{x-2}$ and state the domain and range.

14. **(3 points)** Rewrite $\log_a\left(\dfrac{a^2\sqrt{b}}{c^3}\right)$ as a sum and/or difference of simple logarithms, and simplify.

15. **(6 points)** Solve each of the following equations without the use of calculator.

(a) $\log_4(x-5)+\log_4(x+1)=2$

(b) $3\log_8 x=\log_8 27$.

16. **(6 points)** (a) Solve for x: $5^x=70.5$

(b) Find $\log_3 18$.

17. **(4 points)** If $\tan\theta = \frac{4}{7}$ and $\sin\theta < 0$, then find the exact value of

(a) $\cot\theta$

(b) $\cos\theta$

18. **(3 points)** A 1.9 m tall man casts a shadow of 1.5 m. Find the angle of elevation of the sun.

19. **(3 points)** The angle of elevation from the bottom of a tree to a nearby building is 15°. Find the height of the building if the tree is 55 meters from the base of the building:

20. **(6 points)** Find the exact value of:

(a) $2\cos 30^\circ \cos\frac{\pi}{4} - 4\sin 180^\circ$

(b) $\sin\left(-\frac{7\pi}{6}\right)$

(c) $\tan^{-1}\left(\sqrt{3}\right)$

21. **(4 points)** Graph one cycle of $y = 3\cos\frac{x}{2}$ and state the period and amplitude.

22. **(6 points)** Verify the identities:

(a) $\dfrac{\tan^3\theta\cos^3\theta}{\sin\theta\left(1-\cos^2\theta\right)} = 1$

(b) $2\cos^2\theta - \sin 2\theta\tan\theta = 2\cos 2\theta$.

23. **(3 points)** Solve for x $\left(0 \le x < 2\pi\right)$

$2\cos x + 1 = 0$.

Information Sheet

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$m = \frac{y_2-y_1}{x_2-x_1}$$

parallel lines: $m_1 = m_2$

perpendicular lines: $m_1 m_2 = -1$

vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$(x-h)^2+(y-k)^2 = r^2$

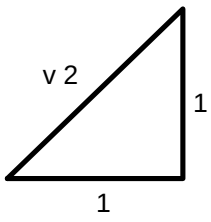
$\log_a(x \cdot y) = \log_a(x)+\log_a(y)$

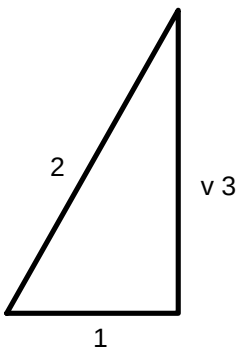
$\log_a\left(\frac{x}{y}\right) = \log_a(x)-\log_a(y)$

$\log_a(x^p) = p\log_a(x)$

$\log_a(1) = 0$ and $\log_a(a) = 1$

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$





SOH CAH TOA

Syr Cxr Tyx $(r^2 = x^2+y^2)$

$\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$\sin^2\theta+\cos^2\theta = 1$

$\tan^2\theta+1 = \sec^2\theta$

$1+\cot^2\theta = \csc^2\theta$

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\text{function}(\theta) = \text{cofunction}(90^\circ-\theta)$

$\sin(A+B) = \sin A\cos B+\cos A\sin B$

$\sin(A-B) = \sin A\cos B-\cos A\sin B$

$\cos(A+B) = \cos A\cos B-\sin A\sin B$

$\cos(A-B) = \cos A\cos B+\sin A\sin B$

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A-\sin^2 A$

$\cos 2A = 2\cos^2 A-1$

$\cos 2A = 1-2\sin^2 A$

If $y = \arcsin x$ then $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

If $y = \arccos x$ then $0 \le y \le \pi$.

If $y = \arctan x$ then $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

ANSWERS

1. quotient $x^2 - x + 2$, remainder 0

2. $(2x-3)(4x^2+6x+9)$

3. $x+3$

4. $\dfrac{b}{a(a-b)}$

5. $3-\sqrt{3}$

6. $x=-4$

7. $x=-2,\ \frac{6}{5}$

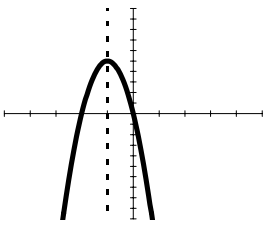
8. $(x+4)^2+(y-5)^2=36$

9. $y=\frac{3}{2}x+5$

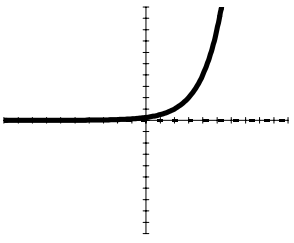
10. **(a)** $\{x\in R \mid x\neq -3\}$ **(b)** $4x+2h$ **(c)** $\dfrac{4}{x}-3$ **(d)** $\frac{1}{2}$

11. **(a)** -1 **(b)** $(2,1)$ **(c)** $4\sqrt{2}$

12. vertex $(-1,5)$
intercepts $(-2,0)$ and $(0,0)$
range $\{y\in R \mid y\leq 5\}$



13. domain $\{x\in R\}$
range $\{y\in R \mid y>0\}$



14. $2+\frac{1}{2}\log_a(b)-3\log_a(c)$

15. **(a)** $x=7$ **(b)** $x=3$

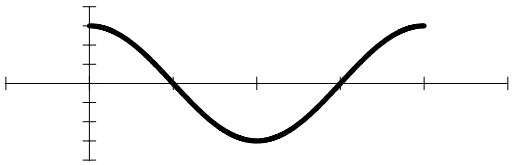
16. (a) $x = 2.644160845$ **(b)** $x = 2.630929754$ **17. (a)** $\frac{7}{4}$ **(b)** $-\frac{7}{\sqrt{65}}$

18. 51.70983681° **19.** 14.73720558

20. (a) $\frac{\sqrt{6}}{2}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{\pi}{3}$

21. The period is 4π.

The amplitude is 3.



22. (a) $LHS = \dfrac{\tan^3\theta\cos^3\theta}{\sin\theta\left(1-\cos^2\theta\right)} = \dfrac{\dfrac{\sin^3\theta}{\cos^3 x}\cdot\cos^3\theta}{\sin\theta\left(\sin^2\theta\right)} = \dfrac{\sin^3\theta}{\sin^3\theta} = 1 = RHS$

(b) $LHS = 2\cos^2\theta - \left(2\sin\theta\cos\theta\right)\cdot\left(\dfrac{\sin\theta}{\cos\theta}\right) = 2\left(\cos^2\theta - \sin^2\theta\right) = 2\cos 2\theta$

23. $x = \frac{2\pi}{3},\ \frac{4\pi}{3}$