

Name: SOLUTIONS

TEST 1 - Version 1
201-009-DW
Functions & Trigonometry
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Question 1 (11 marks)

Simplify the following expressions.

(Note that your answers should be expressed using only positive exponents).

(a) (2 marks) $(-5a^2b^3)(2a^{-2}b^{-4})$
 $(-5)(2)a^{2+(-2)}b^{3+(-4)}$
 $= -10a^0b^{-1}$
 $= \boxed{-10/b}$

(b) (3 marks) $\frac{(2xy^{-3})^{-2}}{(3x^{-2}y^4)^{-3}} = \frac{2^{-2}x^{-2}y^6}{3^{-3}x^6y^{-12}} = \boxed{\frac{3^3y^{18}}{2^2x^8}}$

(c) (3 marks) $\frac{(a^{-2}b^3)^{-4}}{(a^{-3}b^2)^{-2}(ab)^{-4}} = \frac{a^8b^{-12}}{a^6b^{-4}a^{-4}b^{-4}} = \frac{a^8b^{-12}}{a^2b^{-8}} = \boxed{\frac{a^6}{b^4}}$

(d) (3 marks) $\frac{(mn^{-1}p^{-3})^{-1}}{m^0n^{-2}p^3} = \frac{m^{-1}n^1p^3}{n^{-2}p^3} = \frac{1}{mn^{-3}} = \boxed{\frac{n^3}{m}}$

Question 2 (15 marks)

Solve the following equations.

(a) (1 mark) $4(x-1) = x+17$

$$4x-4 = x+17$$

$$3x = 21$$

$$\boxed{x = 7}$$

(b) (2 marks) $x^2 - 2x - 63 = 0$

$$(x-9)(x+7) = 0$$

$$\boxed{x = 9} \text{ or } \boxed{x = -7}$$

(c) (3 marks) $10x^2 - 23x + 12 = 0$

TWO METHODS

① FACTORING

$a \cdot b = 120$

$a+b = -23 \quad -15 \text{ \& } -8$

$$10x^2 - 8x - 15x + 12 = 0$$

$$2x(5x-4) - 3(5x-4) = 0$$

$$(5x-4)(2x-3) = 0$$

So $5x-4=0$ OR $2x-3=0$
 $\boxed{x = 4/5}$ OR $\boxed{x = 3/2}$

(d) (2 marks) $2x^2 = -x-2$

$$2x^2 + x + 2 = 0$$

$$\sqrt{b^2 - 4ac}$$

$$= \sqrt{1 - 4(2)(2)}$$

$$= \sqrt{-15} \text{ impossible, there are no solutions}$$

OR ② QUADRATIC EQUATION

$$x = \frac{23 \pm \sqrt{(-23)^2 - 4(10)(12)}}{20}$$

$$= \frac{23 \pm 7}{20}$$

$$x = \frac{30}{20} = \frac{3}{2} \text{ OR } x = \frac{16}{20} = \frac{4}{5}$$

(e) (3 marks) $4x^3 - 12x^2 + 5x = 0$

$$x(4x^2 - 12x + 5) = 0$$

$$a \cdot b = 20$$

$$x(4x^2 - 10x - 2x + 5) = 0$$

$$a + b = -12$$

$$x(2x(2x-5) - 1(2x-5)) = 0$$

$$-10 \text{ \& } -2$$

$$x(2x-5)(2x-1) = 0$$

$$\boxed{x=0} \text{ OR } \boxed{x=5/2} \text{ OR } \boxed{x=1/2}$$

(f) (2 marks) $x^2 = 13x - 42$

$$x^2 - 13x + 42 = 0$$

$$(x-6)(x-7) = 0$$

$$\boxed{x=6} \text{ OR } \boxed{x=7}$$

(g) (2 marks) $3[2x - (x-2)] = -3(3-2x)$

$$3[x+2] = -9+6x$$

$$3x+6 = -9+6x$$

$$-3x = -15$$

$$\boxed{x=5}$$

Question 3 (3 marks)

Multiply and simplify.

$$\begin{aligned} 7x^3(3x+2)^2 &= 7x^3(3x+2)(3x+2) \\ &= 7x^3(9x^2+12x+4) \\ &= \boxed{63x^5 + 84x^4 + 28x^3} \end{aligned}$$

Question 4 (5 marks)Perform the following division: $(x^4 - 6x^2 + 5x + 4) \div (x - 2)$

$$\begin{array}{r}
 x^3 + 2x^2 - 2x + 1 \\
 x-2 \overline{) x^4 - 6x^2 + 5x + 4} \\
 \underline{-(x^4 - 2x^3)} \\
 2x^3 - 6x^2 + 5x + 4 \\
 \underline{-(2x^3 - 4x^2)} \\
 -2x^2 + 5x + 4 \\
 \underline{-(-2x^2 + 4x)} \\
 x + 4 \\
 \underline{-(x - 2)} \\
 6
 \end{array}$$

SO $\frac{x^4 - 6x^2 + 5x + 4}{x - 2} = \boxed{x^3 + 2x^2 - 2x + 1 + \frac{6}{x - 2}}$

Question 5 (6 marks)

Simplify.

$$\frac{4x^2 - 25}{3x + 3} \cdot \frac{7x - 1}{2x^2 - 9x + 10} \div \frac{2x + 5}{3x^2 - 3x - 6}$$

$$= \frac{\cancel{(2x+5)}\cancel{(2x-5)}}{\cancel{3}(x+1)} \cdot \frac{7x-1}{\cancel{(2x-5)}\cancel{(x-2)}} \cdot \frac{\cancel{3}\cancel{(x-2)}\cancel{(x+1)}}{\cancel{(2x+5)}}$$

$$= \boxed{7x - 1}$$

Question 6 (6 marks)

Add/subtract and simplify.

(a) (4 marks)

$$\begin{aligned} & \frac{x}{1-x} + \frac{2}{x^2-1} \\ &= \frac{-x}{x-1} + \frac{2}{(x-1)(x+1)} \\ &= \frac{-x(x+1)}{(x-1)(x+1)} + \frac{2}{(x-1)(x+1)} \\ &= \frac{-x^2-x+2}{(x-1)(x+1)} = \frac{-(x^2+x-2)}{(x-1)(x+1)} = \frac{-(x+2)(x-1)}{(x-1)(x+1)} \\ &= \boxed{-\frac{(x+2)}{(x+1)}} \end{aligned}$$

(b) (2 marks)

$$\begin{aligned} & \frac{2x^{-1}+4x^{-2}}{2x^{-2}+x^{-1}} \\ &= \frac{\frac{2}{x} + \frac{4}{x^2}}{\frac{2}{x^2} + \frac{1}{x}} = \frac{\frac{2x+4}{x^2}}{\frac{2+x}{x^2}} = \frac{2x+4}{2+x} = \frac{2x+4}{x^2} \cdot \frac{x^2}{2+x} \\ &= \frac{2(x+2)}{(2+x)} \\ &= \boxed{2} \end{aligned}$$

Question 7 (4 marks)

The polynomial $x^5 - 3x^4 - 10x^3 + 30x^2 + 9x - 27$ has five factors. One of them is $(x-3)$. Find the other 4 factors.

Since $x-3$ is a factor then the polynomial can be divided by it

$$\begin{array}{r} x^4 - 10x^2 + 9 \\ x-3 \overline{) x^5 - 3x^4 - 10x^3 + 30x^2 + 9x - 27} \\ \underline{-(x^5 - 3x^4)} \\ -10x^3 + 30x^2 + 9x - 27 \\ \underline{-(-10x^3 + 30x^2)} \\ 9x - 27 \\ \underline{-(9x - 27)} \\ 0 \end{array}$$

So its factors are
 $(x-3)(x^4 - 10x^2 + 9)$
 $(x-3)(x^2 - 9)(x^2 - 1)$
 $(x-3)(x+3)(x-3)(x+1)(x-1)$
 The other 4 factors are listed above.