Name: SOLUTIONS

TEST 2 (201-009-50)
Functions & Trigonometry

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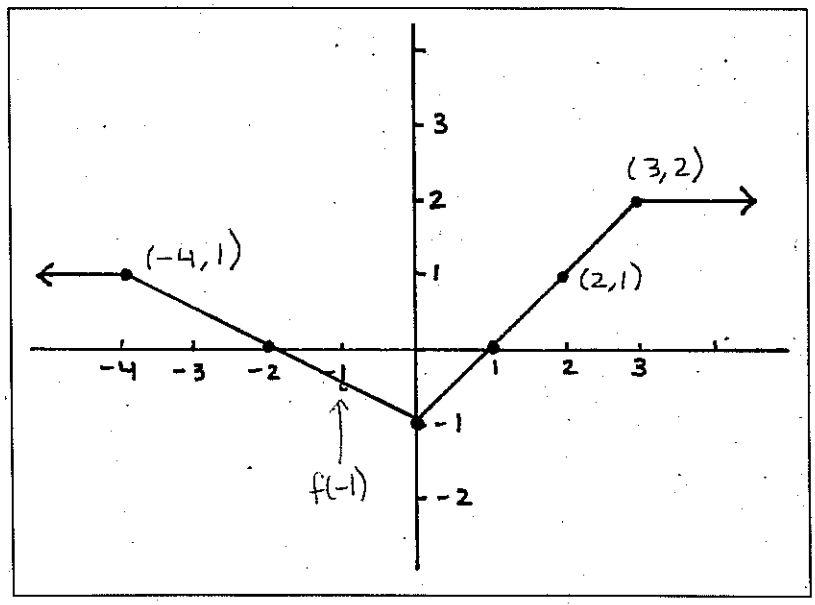
Date: October 16th 2009

This test is marked out of **70 marks**.
Scientific calculator is permitted.
SHOW ALL YOUR WORK.

Question 1 *(7 marks)*



Consider the graph of y = f(x) given above.

(a) Give the domain and range of f
(b) Find f(-1)
(c) Is f(3) = 2 f(2)?
(d) For what values of x is f(x) = 0?

(a) DOMAIN $\mathbb{R}$
RANGE $[-1, 2]$

(b) $f(-1) = -\frac{1}{2}$ (AS SHOWN ON GRAPH)

(c) $f(3) = 2$
$2f(2) = 2(1) = 2$ YES $f(3) = 2f(2)$

(d) $x = -2$ & $x = 1$

Question 2 *(8 marks)*
Simplify the following radical expressions.

(a) *(2 marks)* $3\sqrt{63} - 4\sqrt{28} + \sqrt{700}$

$= 3\sqrt{9 \cdot 7} - 4\sqrt{4 \cdot 7} + \sqrt{100 \cdot 7}$

$= 3 \cdot 3\sqrt{7} - 4 \cdot 2\sqrt{7} + 10\sqrt{7}$

$= 9\sqrt{7} - 8\sqrt{7} + 10\sqrt{7}$

$= \boxed{11\sqrt{7}}$

(b) *(2 mark)* $(2\sqrt{6})(3\sqrt{3})$

$= 6\sqrt{18}$

$= 6\sqrt{9 \cdot 2}$

$= 6 \cdot 3\sqrt{2}$

$= \boxed{18\sqrt{2}}$

(c) *(2 marks)* $(-27)^{2/3} + 27^{2/3} - 16^{3/2}$

*Show your work, do not use a calculator

$= \left(\sqrt[3]{-27}\right)^2 + \left(\sqrt[3]{27}\right)^2 - \left(\sqrt{16}\right)^3$

$= (-3)^2 + 3^2 - 4^3$

$= 9 + 9 - 64$

$= \boxed{-46}$

(d) *(2 marks)* $(3^5)^{-3/5} - [(-3)^6]^{-1/3}$

*Show your work, do not use a calculator

$= 3^{5 \cdot (-\frac{3}{5})} - (-3)^{6 \cdot (-\frac{1}{3})}$

$= 3^{-3} - (-3)^{-2}$

$= \frac{1}{3^3} - \frac{1}{3^2} = \frac{1}{27} - \frac{1}{9} = \frac{1}{27} - \frac{3}{27} = \boxed{\frac{-2}{27}}$

Question 3 *(4 marks)*
Rationalize the denominator and simplify.

$$\frac{7+3\sqrt{5}}{2-\sqrt{5}}$$

$$= \frac{7+3\sqrt{5}}{2-\sqrt{5}} \left(\frac{2+\sqrt{5}}{2+\sqrt{5}}\right)$$

$$= \frac{14+7\sqrt{5}+6\sqrt{5}+3\cdot 5}{4-5}$$

$$= \frac{29+13\sqrt{5}}{-1} = \boxed{-29-13\sqrt{5}}$$

Question 4 *(2 marks)*
Perform the multiplication and simplify.

$(4\sqrt{5}-2\sqrt{2})(2\sqrt{5}+3\sqrt{2})$

$$= 8\cdot 5 + 12\sqrt{5}\sqrt{2} - 4\sqrt{2}\sqrt{5} - 6\cdot 2$$

$$= 40 + 12\sqrt{10} - 4\sqrt{10} - 12$$

$$= \boxed{28+8\sqrt{10}}$$

Question 5 *(9 marks)*

If $f(x) = 2x^2 - 1$ and $g(x) = -x + 3$ then find the following:

(a) *(1 marks)* $f(-3) + g(2)$

$$f(-3) + g(2) = [2(-3)^2 - 1] + [-(2) + 3]$$
$$= [17] + [1]$$
$$= \boxed{18}$$

(b) *(1.5 marks)* $f(a^2) + g(a^4)$

$$= [2(a^2)^2 - 1] + [-a^4 + 3]$$
$$= 2a^4 - 1 - a^4 + 3$$
$$= \boxed{a^4 + 2}$$

(c) *(2.5 marks)* $(f \circ g)(x)$

$$f(g(x)) = f(-x + 3)$$
$$= 2(-x + 3)^2 - 1$$
$$= 2(x^2 - 6x + 9) - 1$$
$$= \boxed{2x^2 - 12x + 17}$$

(d) *(2 marks)* $(g \circ f)(x)$

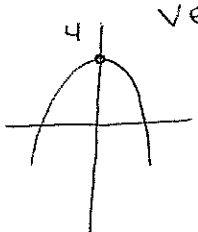
$$g(f(x)) = g(2x^2 - 1)$$
$$= -(2x^2 - 1) + 3$$
$$= \boxed{-2x^2 + 4}$$

(e) *(2 marks)* the *domain and the range of* **g o f**

$g \circ f$ is a parabola with vertex:

$$x = \frac{0}{2(-2)} = 0 \qquad y = -2(0)^2 + 4 = 4$$

vertex $(0, 4)$



domain $\mathbb{R}$
range $(-\infty, 4]$

Question 6 *(7 marks)*
Find the domain of the following functions

(a) *(1 marks)* $f(x) = \sqrt{1-x}$

$(-\infty, 1]$

(b) *(1.5 marks)* $f(x) = \dfrac{4}{\sqrt{1-x}}$

$(-\infty, 1)$

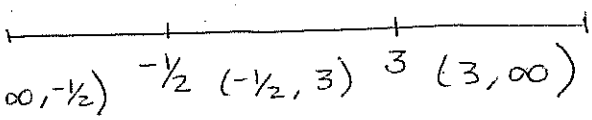
(d) *(2 marks)* $f(x) = \dfrac{1}{2x^2 - 5x - 3}$

$= \dfrac{1}{2x^2-6x+x-3} = \dfrac{1}{2x(x-3)+1(x-3)}$

$= \dfrac{1}{(2x+1)(x-3)}$

DOMAIN: $\mathbb{R} \setminus \{-\frac{1}{2}, 3\}$

(d) *(2.5 marks)* $f(x) = \dfrac{1}{\sqrt{2x^2 - 5x - 3}}$



$(-\infty, -\frac{1}{2})$ $(-\frac{1}{2}, 3)$ $(3, \infty)$

TEST POINTS $x = -1$ $x = 0$ $x = 4$

$f(-1) = \dfrac{1}{\sqrt{4}} = \frac{1}{2}$ $f(0) = \dfrac{1}{\sqrt{-3}}$ $f(4) = \dfrac{1}{\sqrt{9}} = \frac{1}{3}$

DOMAIN MUST EXCLUDE $(-\frac{1}{2}, 3)$ & $-\frac{1}{2}$ & 3

DOMAIN IS $(-\infty, -\frac{1}{2}) \cup (3, \infty)$

Question 7 *(5 marks)*

Perform the indicated operations and simplify.

$$\frac{5a^2}{a^2-4} + \frac{3}{2-a} = \frac{5a-1}{a+2}$$

$$\frac{5a^2}{(a+2)(a-2)} + \frac{3}{-(a-2)} = \frac{5a-1}{a+2}$$

$$\frac{5a^2}{(a+2)(a-2)} - \frac{3(a+2)}{(a-2)(a+2)} = \frac{(5a-1)(a-2)}{(a+2)(a-2)}$$

$$\frac{5a^2-3a-6}{(a+2)(a-2)} = \frac{5a^2-11a+2}{(a+2)(a-2)}$$

$$\frac{(5a^2-3a-6)-(5a^2-11a+2)}{(a+2)(a-2)} = \frac{0}{(a+2)(a-2)}$$

$$-3a-6+11a-2=0$$

$$8a-8=0$$

$$8a=8$$

$$\boxed{a=1}$$

Question 8 *(4 marks)*

Find the equation of the line passing through the point (2, –1) that is perpendicular to the line $4y-2x=6$.

$$4y = 2x+6$$

$$y = \frac{2}{4}x + \frac{6}{4} = \frac{1}{2}x + \frac{3}{2}$$

slope is negative reciprocal $m=-2$

$$y = -2x + b$$

plug in $(2,-1)$

$$-1 = -2(2) + b$$

$$-1 = -4 + b$$

$$3 = b$$

$$\boxed{y = -2x + 3}$$

Question 9 *(6 marks)*

The height h of a ball t seconds after being thrown is given by the equation:

$$h(t) = -8.5t^2 + 25t + 4.5$$

(a) *(1 mark)* What is the height of the ball at the moment of its release?

(b) *(2.5 marks)* After how many seconds is the ball's height less than when it was released?

(c) *(2.5 marks)* What is the maximum height that the ball reaches?

(a) $t = 0 \quad h(0) = 4.5$

(b) $h(t) = 4.5$ so $4.5 = -8.5t^2 + 25t + 4.5$

$$0 = -8.5t^2 + 25t$$

$$t = \frac{-25 \pm \sqrt{(25)^2 - 4(-8.5)(0)}}{2(-8.5)}$$

$$= \frac{-25 \pm 25}{-17} \Rightarrow t = 0 \quad t = \frac{-50}{-17}$$

$$\boxed{t = 50/17}$$

(c) vertex $t = \frac{-25}{2(-8.5)} = 25/17$ sec

Height $h\left(25/17\right) = -8.5\left(25/17\right)^2 + 25\left(25/17\right) + 4.5$

$$= 389/17 = \boxed{22.88\text{ m}}$$

Question 10 *(4 marks)*

Find the value of the variable k, if the slope of the line passing through the points (k, 15) and (2, 3) is equal to 4.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow 4 = \frac{15-3}{K-2}$$

$$4 = \frac{12}{K-2}$$

$$4(K-2) = 12$$

$$4K - 8 = 12$$

$$4K = 20$$

$$\boxed{K = 5}$$

Question 11 *(6 marks)*

Sketch the graph of the parabola $y = -x^2 - 2x - 3$. Your sketch should contain the following points, clearly indicated: vertex, x-intercepts, y-intercept. You should also indicate the domain and the range of the parabola.

Vertex $\quad x = \frac{-(-2)}{2(-1)} = -1$

$y = -(-1)^2 - 2(-1) - 3$

$= -1 + 2 - 3$

$= -2 \qquad (-1, -2)$

Y-INTERCEPT $\quad (0, -3)$

X-INTERCEPTS $\quad 0 = -x^2 - 2x - 3$

$x^2 + 2x + 3 = 0$

$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(3)}}{2}$

$= \frac{-2 \pm \sqrt{-8}}{2}$ NO INTERCEPTS

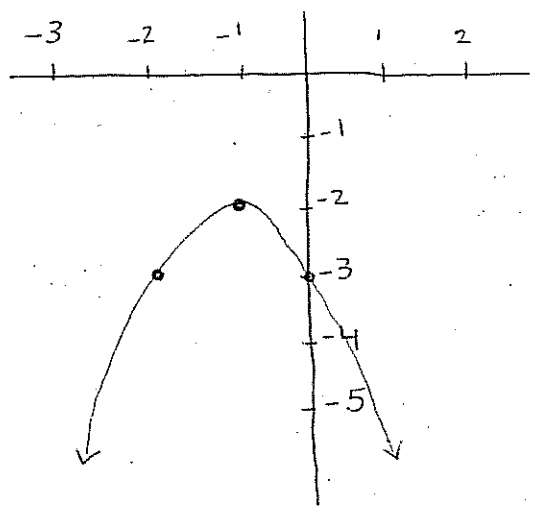
ANOTHER POINT ON THE GRAPH

$x = -2 \qquad y = -(-2)^2 - 2(-2) - 3$

$= -4 + 4 - 3$

$= -3$

$(-2, -3)$



DOMAIN $\quad \mathbb{R}$

RANGE $\quad (-\infty, -2]$

Question 12 *(8 marks)*

Given the functions $f(x) = \frac{2}{x}$ $\quad g(z) = -z^2 - 1$ $\quad h(t) = t^2 - t + 2$

Find the following:

(a) $(f \circ h)(2x)$
(b) $(h \circ g)(t)$
(c) $(g \circ h \circ f)(-1)$
(d) $(f \circ h)(-1) + (h \circ g)(2)$

(a) $f(h(2x)) = f((2x)^2 - 2x + 2)$
$= f(4x^2 - 2x + 2)$
$= \boxed{\frac{2}{4x^2 - 2x + 2}}$

(b) $h(g(t)) = h(-t^2 - 1) = (-t^2 - 1)^2 - (-t^2 - 1) + 2$
$= t^4 + 2t^2 + 1 + t^2 + 1 + 2$
$= \boxed{t^4 + 3t^2 + 4}$

(c) $(g \circ h)(f(-1))$
$= (g \circ h)\left(\frac{2}{-1}\right) = g(h(-2))$
$= g((-2)^2 - (-2) + 2) = g(8) = -8^2 - 1 = \boxed{-65}$

(d) $f(h(-1))$
$= f(4)$
$= 2/4 = 1/2$

$h(g(2))$
$= h(-2^2 - 1)$
$= h(-5)$
$= (-5)^2 - (-5) + 2 = 32$

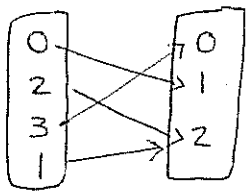
$(f \circ h)(-1) + (h \circ g)(2)$
$= 1/2 + 32$
$= \boxed{32.5}$

BONUS *(3 marks)*

Which of the following functions are injective? Explain your answer.

(a) { (0,1), (2,2), (3,0), (1,2) }
(b) $f(x) = x^2 - 1$
(c) $g(x) = \sqrt{x}$
(d) $h(t) = 2t - 3$

(a)



DOM. RANGE

NOT INJECTIVE BOTH 1 & 2 ARE SENT TO 2

(b) $X^2 - 1$

NOT INJECTIVE both 1 & -1 are SENT TO 0 FOR EXAMPLE

(c) INJECTIVE; EACH $\sqrt{x}$ COMES FROM only one X VALUE

(d) $h(t) = 2t - 3$ is a line EACH value $h(t)$ comes FROM only ONE t-value.