Name: SOLUTIONS

TEST 3 (201-009-DW)
Functions & Trigonometry
Instructor: Emilie Richer
Date: November 26th 2009

This test is marked out of **60 marks.**
Scientific calculator is permitted.
SHOW ALL YOUR WORK.

Question 1 *(5 marks)*
(a) Find the inverse of the function $f(x) = \frac{x+2}{x}$
(b) Briefly explain how you would check if your solution is in fact the inverse (you do not have to explicitly check, just explain how you would do so)

(a) $x = \frac{y+2}{y}$

$xy = y+2$

$xy - y = 2$

$y(x-1) = 2$

$y = \frac{2}{x-1}$

so $\boxed{f^{-1}(x) = \frac{2}{x-1}}$

(b) We could check if this was in fact the inverse of f by taking the "composition" $f^{-1}\circ f(x)$ $f\circ f^{-1}(x)$ & seeing if it gives x.

Question 2 *(3 marks)*
Solve the following equation for x
$\ln(x + 1) + \ln x = \ln 2$

$\ln\left((x+1)x\right) = \ln 2$

$x^2 + x = 2$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2$ or $x = 1$

But only $\boxed{x = 1}$ is a possible solution.

Question 3 *(4 marks)*
Solve the following equation for x
$\log_3(2x+1) + \log_3(x-2) = 2$

$\log_3((2x+1)(x-2)) = 2$

$3^2 = (2x+1)(x-2)$

$9 = 2x^2 - 3x - 2$

$2x^2 - 3x - 11 = 0$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-11)}}{2(2)} = \frac{3 \pm \sqrt{97}}{4}$$

BUT ONLY $\boxed{x = \frac{3+\sqrt{97}}{4}}$ is A solution

Question 4 *(4 marks)*
Solve the following equation for x
$2(5^{x+1}) = 3^{1-x}$

$\ln(2 \cdot 5^{x+1}) = \ln(3^{1-x})$

$\ln 2 + (x+1)\ln 5 = (1-x)\ln 3$

$\ln 2 + \ln 5\, x + \ln 5 = \ln 3 - \ln 3\, x$

$\ln 5\, x + \ln 3\, x = \ln 3 - \ln 2 - \ln 5$

$x(\ln 5 + \ln 3) = \ln 3 - \ln 2 - \ln 5$

$$\boxed{x = \frac{\ln 3 - \ln 2 - \ln 5}{\ln 5 + \ln 3}}$$

$$\boxed{x = -0.44}$$

Question 5 *(5 marks)*
Solve the following equation for x
$7^{2x} - 4(7^x) + 3 = 0$

$$(7^x)^2 - 4(7^x) + 3 = 0 \qquad \text{Let } y = 7^x$$

QUADRATIC $\quad y^2 - 4y + 3 = 0$

$$(y-3)(y-1) = 0 \qquad y = 3 \text{ OR } y = 1$$

SO

$$7^x = 3 \qquad \text{OR} \qquad 7^x = 1$$

$$\ln 7^x = \ln 3 \qquad \ln 7^x = \ln 1$$

$$x \ln 7 = \ln 3 \qquad x \ln 7 = 0$$

$$\boxed{x = \frac{\ln 3}{\ln 7}} \qquad \boxed{x = 0}$$

Question 6 *(3 marks)*
Convert the following values into degrees
(a) $\frac{5\pi}{4}$
(b) 12π
(c) $\frac{11\pi}{7}$

(a) $\Rightarrow \quad \frac{5\pi}{4} \cdot \frac{180}{\pi} = 225°$

(b) $\Rightarrow \quad 12\pi = 12\pi\left(\frac{180}{\pi}\right) = 2160°$

(c) $\Rightarrow \quad \frac{11\pi}{7}\left(\frac{180}{\pi}\right) = 282.86°$

Question 6 *(6 marks)*

(a) Find the inverse of the function $f(x) = \log_2(x + 1)$

(b) Sketch the function f as well as its inverse f^{-1} on the same graph

(c) State the domain and range of the functions f and f^{-1}

(a) $y = \log_2(x+1)$

$x = \log_2(y+1)$

$2^x = y+1$

$y = 2^x - 1$

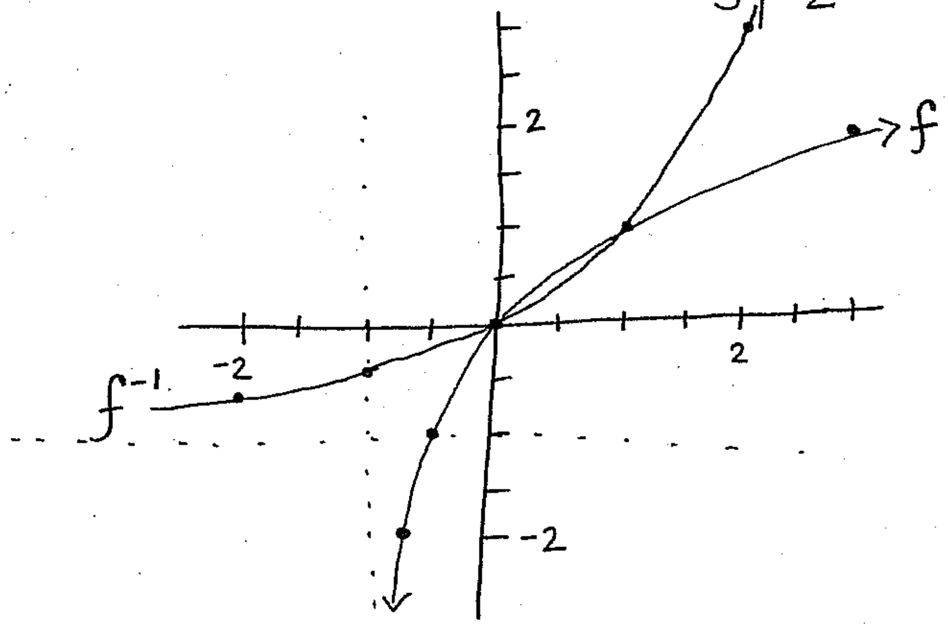
$\boxed{f^{-1}(x) = 2^x - 1}$

(b) $f^{-1}(x) = 2^x - 1$

x	y
-2	$-3/4$
-1	$-1/2$
0	0
1	1
2	3

$f(x) = \log_2(x+1)$

x	y
$-3/4$	-2
$-1/2$	-1
0	0
1	1
3	2



(c) DOMAIN (f^{-1}) $\mathbb{R}$

RANGE (f^{-1}) $(-1, \infty)$

DOMAIN (f) $(-1, \infty)$

RANGE (f) $\mathbb{R}$

Question 7 *(6 marks)*

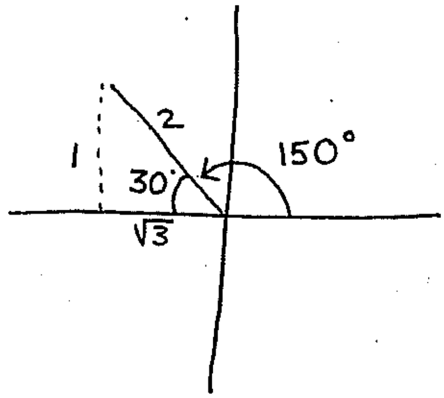
Find the exact value (no decimals) of the following

(a) sec (150°)

(b) cot (240°)

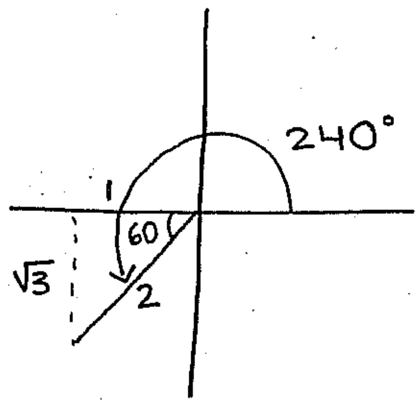
(c) sin (− 855°)

(a)



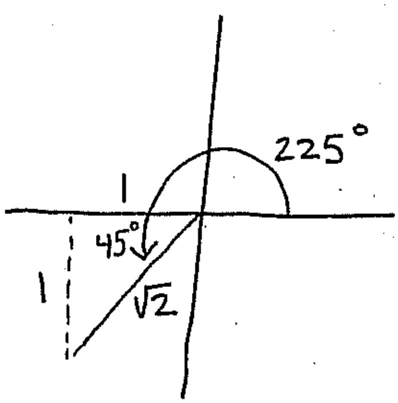
$$\sec(150°) = \frac{r}{x} = \boxed{\frac{2}{-\sqrt{3}}}$$

(b)



$$\cot(240°) = \frac{x}{y} = \frac{-1}{-\sqrt{3}} = \boxed{\frac{1}{\sqrt{3}}}$$

(c)

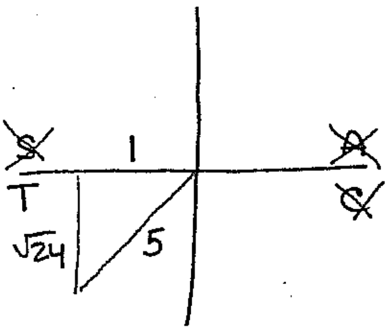


$$\sin(-855°) = \sin(225°)$$

$$\sin(-855°) = \frac{y}{r} = \boxed{\frac{-1}{\sqrt{2}}}$$

Question 8 *(5 marks)*

Find the **exact** values of the five other trigonometric functions if $\sec\theta = -5$ and $\csc\theta < 0$



$\sec\theta = \frac{r}{x} = -5$ so $r = 5$, $x = -1$

$x^2 + y^2 = r^2$

$1 + y^2 = 25 \longrightarrow y^2 = 24$

$y = \pm\sqrt{24}$

$y = -\sqrt{24}$

(b/c of position of angle)

$\sin\theta = \frac{y}{r} = \frac{-\sqrt{24}}{5}$ $\quad\cos\theta = \frac{1}{-5}$

$\csc\theta = \frac{r}{y} = \frac{-5}{\sqrt{24}}$ $\quad\tan\theta = \sqrt{24}$ $\quad\cot\theta = \frac{1}{\sqrt{24}}$

Question 9 *(5 marks)*

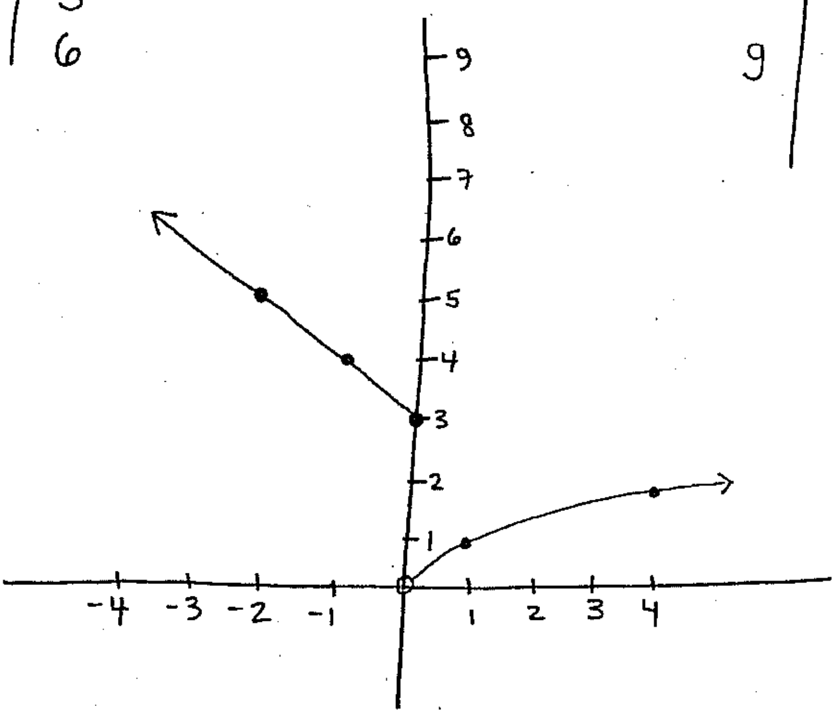
Sketch the piece-wise function given by $f(x) = \begin{cases} -x+3 & \text{if } x \le 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$

$x \le 0$ $(y = -x+3)$

x	y
0	3
-1	4
-2	5
-3	6

$x > 0$ $(y = \sqrt{x})$

x	y
1	1
4	2
9	3

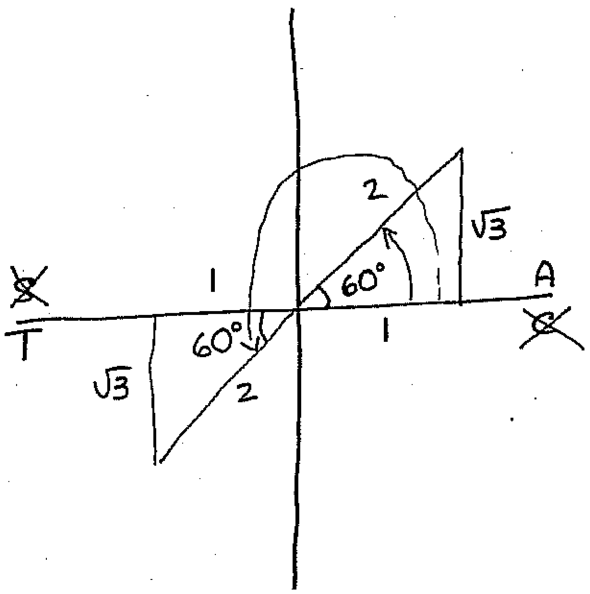


Question 10 *(6 marks)*

Solve for θ giving exact solutions (in radians).

Your solutions should be in the range $0 \le \theta < 2\pi$.

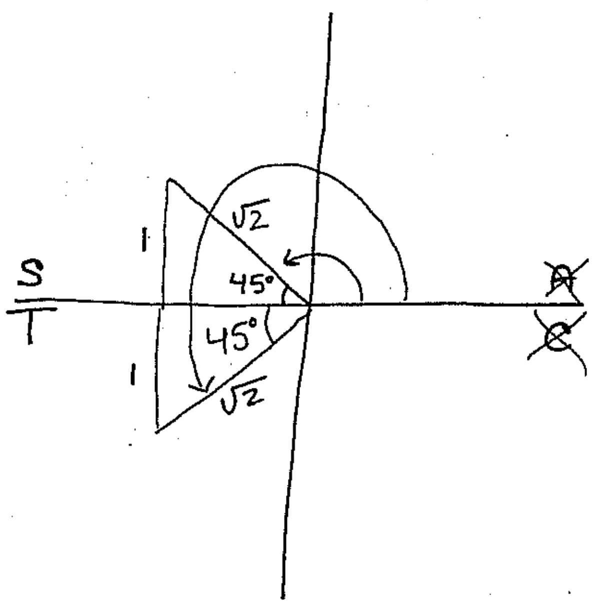
(a) $\tan\theta - \sqrt{3} = 0$ $\tan\theta = \sqrt{3}$



$\theta = 60° = \boxed{\frac{\pi}{3}}$

$\theta = 240° = \boxed{\frac{4\pi}{3}}$

(b) $\sec\theta = -\sqrt{2}$



$\theta = 135° = \boxed{\frac{3\pi}{4}}$

$\theta = 225° = \boxed{\frac{5\pi}{4}}$

Question 11 *(6 marks)*

(a) Find the inverse of the function $f(x) = x^2 - 2, x \leq 0$

(b) Sketch the function f as well as its inverse f^{-1} on the same graph

(c) State the domain and range of the functions f and f^{-1}

(a)

$$x = y^2 - 2 \qquad y \leq 0$$

$$y^2 = x + 2$$

$$y = -\sqrt{x+2}$$

$$\boxed{f^{-1}(x) = -\sqrt{x+2}}$$

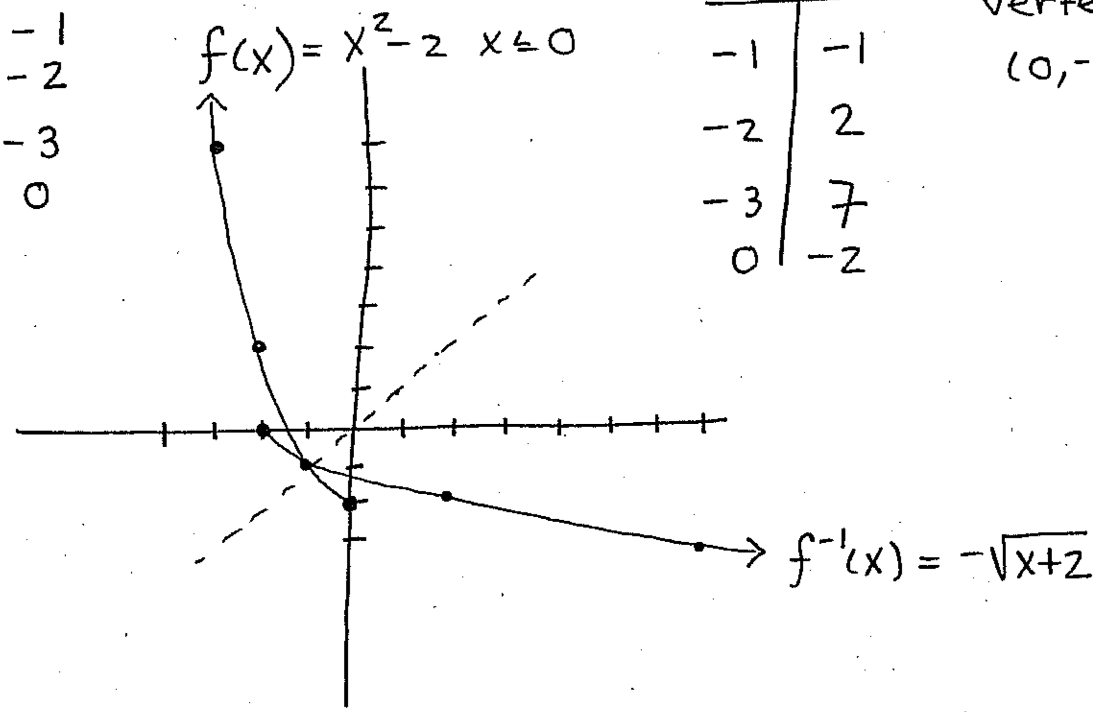
(b) $f^{-1}(x) = -\sqrt{x+2}$

x	y
-1	-1
2	-2
7	-3
-2	0

$f(x) = x^2 - 2 \quad x \geq 0$

x	y
-1	-1
-2	2
-3	7
0	-2

vertex $(0,-2)$



(c) f^{-1}: DOMAIN $[-2, \infty)$

RANGE $(-\infty, 0]$

f: DOMAIN $(-\infty, 0]$

RANGE $[-2, \infty)$