SOLUTIONS
ASSIGNMENT #6
20th OF OCT 2009
201-914-DW

P. 167

#5 $p = q^2 + 8q + 16$ supply
$p = -3q^2 + 6q + 436$ demand

AT MARKET EQUILIBRIUM
supply = demand

$$q^2 + 8q + 16 = -3q^2 + 6q + 436$$

$$4q^2 + 2q - 420 = 0$$

$$q = \frac{-2 \pm \sqrt{2^2 - 4(4)(-420)}}{2(4)}$$

$$= \frac{-2 \pm \sqrt{6724}}{8} = \frac{-2 \pm 82}{8}$$

$$q = -10.5 \qquad q = 10$$

WE CANNOT HAVE A NEGATIVE QUANTITY
SO $q = 10$

$$p = q^2 + 8q + 16 = 10^2 + 8(10) + 16 = 196$$

MARKET equilibrium is $\boxed{(10, 196)}$.

#12 Supply $2p - q = 50$
demand $pq = 100 + 20q$

Supply $2p - 50 = q$
substitute $q = 2p - 50$ into demand

$$p(2p-50) = 100 + 20(2p-50)$$
$$2p^2 - 50p = 100 + 40p - 1000$$
$$2p^2 - 90p + 900 = 0$$
$$p^2 - 45p + 450 = 0$$
$$p = \frac{45 \pm \sqrt{(-45)^2 - 4(1)(450)}}{2(1)}$$
$$= \frac{45 \pm 15}{2}$$

$p = 30$ or $p = 15$

When $p = 30$
$$q = 2p - 50$$
$$q = 2(30) - 50$$
$$q = 10$$
so $(10, 30)$ is a market equilibrium

When $p = 15$
$$q = 2p - 50$$
$$q = 2(15) - 50$$
$$= 30 - 50$$
$$= -20$$
impossible, no market equilibrium at $p = 15$

There is only one market equilibrium; $\boxed{(10, 30)}$

#19 PROFIT $P(x) = 11.5x - 0.1x^2 - 150$

BREAK EVEN POINT OCCURS WHEN
PROFIT $= 0$

$$0 = 11.5x - 0.1x^2 - 150$$

$$x = \frac{-11.5 \pm \sqrt{(11.5)^2 - 4(-0.1)(-150)}}{2(-0.1)}$$

$$= \frac{-11.5 \pm 8.5}{-0.2} \qquad x = 100 \quad \text{or} \quad x = 15$$

Since NO more than 75 UNITS CAN be produced, break-even point is AT $\boxed{x=15}$.

#28 $P(x) = 50x - 0.2x^2 - 2000$

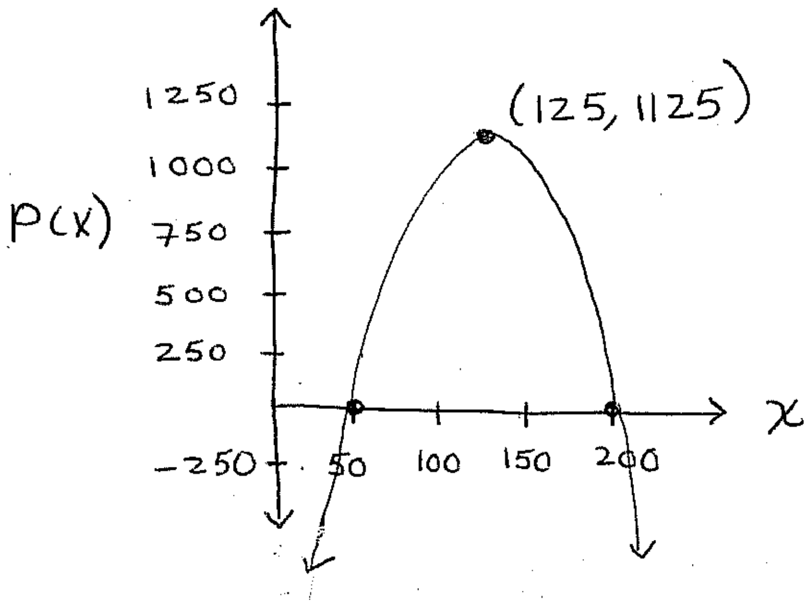
SKETCH: vertex : $\frac{-50}{2(-0.2)} = 125 \qquad x = 125$

y-value vertex : $p(125) = 50(125) - 0.2(125)^2 - 200$
$= 1125$

vertex $(125, 1125)$
y-intercept $(0, -2000)$
x-intercepts $0 = 50x - 0.2x^2 - 2000$

$$x = \frac{-50 \pm \sqrt{50^2 - 4(-0.2)(-2000)}}{2(-0.2)}$$

$$= \frac{-50 \pm 30}{-0.4} \qquad x = 50 \qquad x = 200$$



- MAXIMUM PROFIT 1125 \$ AT 125 ITEMS
- PROFIT WHEN $50 < x < 200$
- LOSS WHEN $0 \leq x < 50$ & $x > 200$
- BREAK EVEN AT $x = 50$, $x = 200$

#31 $C(x) = \left(\frac{2}{5}x + 222\right)x + 28000$

$$= \frac{2}{5}x^2 + 222x + 28000$$

$$R(x) = x \cdot p$$

$$= x\left(1250 - \frac{3}{5}x\right)$$

$$= 1250x - \frac{3}{5}x^2$$

(a) BREAK EVEN points

$$C(x) = R(x)$$

$$\frac{2}{5}x^2 + 222x + 28000 = 1250x - \frac{3}{5}x^2$$

$$x^2 - 1028x + 28000 = 0$$

$$x = \frac{1028 \pm \sqrt{(-1028)^2 - 4(-1)(28000)}}{2(1)}$$

$$= \frac{1028 \pm 972}{2} \qquad \boxed{x = 1000 \quad \text{OR} \quad x = 28}$$

TWO BREAK EVEN POINTS.

(b) MAX Revenue

(VERTEX OF Revenue FUNCTION; since $R(x)$ is concave down PARAbola)

$$R(x) = 1250x - \frac{3}{5}x^2$$

$$x = \frac{-b}{2a} = \frac{-1250}{2(-3/5)} = \frac{6250}{6} = 1041.67$$

$$y = 1250(1041.67) - \frac{3}{5}(1041.67)^2$$

$$= \boxed{651041.67\ \$}$$

is MAXIMUM Revenue

(c) $P(x) = R(x) - C(x)$

$$P(x) = \left(1250x - \frac{3}{5}x^2\right) - \left(\frac{2}{5}x^2 + 222x + 28000\right)$$

$$\boxed{P(x) = -x^2 + 1028x - 28000}$$

MAX PROFIT OCCURS AT VERTEX: $x = \frac{-b}{2a} = \frac{-1028}{2(-1)}$

$$= 514$$

$$P(514) = -(514)^2 + 1028(514) - 28000$$

$$= 236196$$

MAX profit is $\boxed{\$236196}$ WHEN $x = 514$ are produced & sold.

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$$C(x) = \left(\tfrac{3}{4}x + 1460\right)x + 300$$
$$= \tfrac{3}{4}x^2 + 1460x + 300$$
$$R(x) = \left(1500 - \tfrac{1}{4}x\right)x$$
$$= 1500x - \tfrac{1}{4}x^2$$

(a) $C(x) = R(x)$

$$\tfrac{3}{4}x^2 + 1460x + 300 = 1500x - \tfrac{1}{4}x^2$$
$$x^2 - 40x + 300 = 0$$
$$(x-30)(x-10) = 0$$

BREAK even At $\boxed{x=30}$ & $\boxed{x=10}$

(b) MAX Revenue occurs at vertex OF $R(x) = 1500x - \frac{1}{4}x^2$

$$x = \frac{-b}{2a} = \frac{-1500}{2(-1/4)} = 3000$$
$$R(3000) = 1500(3000) - 1/4\,(3000)^2$$
$$= 2,25\,0,000$$

MAX Revenue is 2 250 000 \$

(c) $P(x) = R(x) - C(x)$

$$= 1500x - 1/4\,x^2 - \left(\tfrac{3}{4}x^2 + 1460x + 300\right)$$
$$\boxed{P(x) = -x^2 + 40x - 300}$$

vertex $x = \dfrac{-40}{2(-1)} = 20$

$$P(20) = -(20)^2 + 40(20) - 300 = 100$$

MAX PROFIT is \$100 (WHEN 20 UNITS ARE sold)