

TEST 1 (914-201-DW)**Business Mathematics**

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This test is marked out of marks 60 marks.

You can earn up to 63 marks by completing the bonus question.

Scientific calculator is permitted.

Show all your work.

Question 1 (9 marks)

Simplify the following expressions so that only positive exponents remain

(a) (3 marks) $(-8a^{-3}b^2)(2a^5b^{-4})(2^{-3}ab^{-4})$

$$\left(\frac{-8b^2}{a^3}\right)\left(\frac{2a^5}{b^4}\right)\left(\frac{a}{2^3b^4}\right) = \frac{-16a^6b^2}{8a^3b^8}$$

$$= \boxed{\frac{-2a^3}{b^6}}$$

(b) (3 marks) $\frac{(-2a^{-3}b^2c)^3}{(2a^{-5}b^4c)^{-1}} = \frac{(-2)^3 a^{-9} b^6 c^3}{2^{-1} a^5 b^{-4} c^{-1}}$

$$= \frac{-8 b^0 c^4}{2^{-1} a^{14}} = \boxed{\frac{-16 b^0 c^4}{a^{14}}}$$

(c) (3 marks) $\frac{(2x^{-3}y^2)^{-3}(4xy)^{-2}}{(x^2y)^0(x^2y^{-3})^4} = \frac{2^{-3}x^9y^{-6}4^{-2}x^{-2}y^{-2}}{x^8y^{-12}}$

$$= \frac{x^7y^{-8}}{2^34^2y^{-12}} = \boxed{\frac{x^7y^4}{128}}$$

Question 2 (6 marks)

Simplify the following expressions so that only positive exponents remain.

(a) (1.5 marks) $\frac{-2^{-1}x^{-2}}{(2x)^2} = \frac{-1}{2^1x^2 2^2x^2} = \frac{-1}{2^3x^4} = \boxed{\frac{-1}{8x^4}}$

(b) (1.5 marks) $\frac{(2x^{-2})^{-1}}{(-2x)^{-2}} = \frac{2^{-1}x^2}{(-2)^{-2}x^{-2}} = \frac{(-2)^2x^2}{2x^{-2}} = \boxed{-2x^4}$

(c) (1.5 marks) $\frac{(2^{-1}x^{-2})}{2x^2} = \frac{1}{2^1x^2 2x^2} = \boxed{\frac{1}{4x^4}}$

(d) (1.5 marks) $\frac{(-2x^{-2})^{-1}}{-2x^2} = \frac{(-2)^{-1}x^2}{-2x^2} = \frac{1}{(-2)(-2)^1} = \boxed{\frac{1}{4}}$

Question 3 (15 marks)

Perform the indicated operations and simplify your answer.

(a) (2 marks) $- (8 - 4(q + 5) + q)$

$$= - (8 - 4q - 20 + q)$$

$$= - (-12 - 3q)$$

$$= \boxed{12 + 3q}$$

(b) (3 marks) $y^3 - [y^2 - (y^3 + y^2)] - [y^3 + (1 - y^2)]$

$$= y^3 - [-y^3] - [y^3 + 1 - y^2]$$

$$= y^3 + y^3 - y^3 - 1 + y^2$$

$$= \boxed{y^3 + y^2 - 1}$$

(c) (3 marks) $2x^3(4x + 1)^2$

$$= 2x^3(4x + 1)(4x + 1)$$

$$= 2x^3(16x^2 + 8x + 1)$$

$$= \boxed{32x^5 + 16x^4 + 2x^3}$$

(d) (3 marks) $(2x - 3)(x + 2) - (3x + 5) + 2$

$$= 2x^2 + x - 6 - 3x - 5 + 2$$

$$= \boxed{2x^2 - 2x - 9}$$

(e) (4 marks) $(x^4 + 5x + 3) \div (x - 1)$

(Express your answer in one of the two ways shown in class)

$$\begin{array}{r} x^3 + x^2 + x + 4 \\ x-1 \overline{) x^4 + 5x + 3} \\ \underline{-(x^4 - x^3)} \\ x^3 + 5x + 3 \\ \underline{-(x^3 - x^2)} \\ x^2 + 5x + 3 \\ \underline{-(x^2 - x)} \\ 4x + 3 \\ \underline{-(4x - 4)} \\ 7 \end{array}$$

So $(x^4 + 5x + 3) \div (x - 1) = x^3 + x^2 + x + 4 + \frac{7}{x-1}$

Question 4 (5 marks)

Simplify completely.

$$\frac{x^4 - 2x^3}{3x^2 - x - 2} \div \frac{x^3 - 4x}{9x^2 - 4}$$

$$\frac{x^3(x-2)}{(3x+2)(x-1)} \cdot \frac{(3x-2)(3x+2)}{x(x^2-4)}$$

$$= \frac{x^2(x-2)(3x-2)}{(x-1)(x^2-4)}$$

$$= \frac{x^2(x-2)(3x-2)}{(x-1)(x+2)(x-2)}$$

$$= \boxed{\frac{x^2(3x-2)}{(x-1)(x+2)}}$$

Question 5 (5 marks)

Perform the indicated operations and simplify.

$$\frac{b-1}{b^2+2b} + \frac{b}{3b+6}$$

$$= \frac{b-1}{b(b+2)} + \frac{b}{3(b+2)} \quad \text{Lcd } 3b(b+2)$$

$$= \frac{3(b-1)}{3b(b+2)} + \frac{b^2}{3b(b+2)} = \frac{3b-3+b^2}{3b(b+2)}$$

$$= \boxed{\frac{b^2+3b-3}{3b(b+2)}}$$

Question 6 (5 marks)

Simplify completely.

$$\frac{x^2-5x-6}{x^2-5x+4} \cdot \frac{x^2-x-12}{x^3-6x^2} \cdot \frac{-x^3+x}{x^2-2x+1}$$

$$= \frac{(x-)(x+1)}{(x-4)(x-1)} \cdot \frac{(x-4)(x+3)}{x^2(x-6)} \cdot \frac{(-x)(x^2-1)}{(x-1)(x-1)}$$

$$= \frac{- (x+1)(x+3)(x+1)(x-1)}{(x-1)x(x-1)(x-1)}$$

$$= \boxed{\frac{- (x+1)^2(x+3)}{x(x-1)^2}}$$

Question 7 (5 marks)

Perform the indicated operations and simplify.

$$\frac{x+2}{x^2-x} - \frac{x^2+4}{x^2-2x+1} + 1$$

$$= \frac{x+2}{x(x-1)} - \frac{x^2+4}{(x-1)(x-1)} + 1 \quad \text{Lcd } x(x-1)^2$$

$$= \frac{(x+2)(x-1)}{x(x-1)^2} - \frac{(x^2+4)x}{x(x-1)^2} + \frac{x(x-1)^2}{x(x-1)^2}$$

$$= \frac{(x^2+x-2) - (x^3+4x) + x^3-2x^2+x}{x(x-1)^2}$$

$$= \boxed{\frac{-x^2-2x-2}{x(x-1)^2}}$$

Question 8 (5 marks)

Perform the indicated operations and simplify.

$$\frac{x-1 - \frac{x-1}{x}}{\frac{1}{x-1} + 1}$$

$$= \frac{\frac{x^2}{x} - \frac{1}{x} - \frac{x-1}{x}}{\frac{1}{x-1} + \frac{x-1}{x-1}} = \frac{\frac{x^2-x}{x}}{\frac{x}{x-1}}$$

$$= \frac{x^2-x}{x} \cdot \frac{x-1}{x}$$

$$= \frac{x(x-1)(x-1)}{x^2} = \boxed{\frac{(x-1)^2}{x}}$$

Question 9 (5 marks)

Solve the equation for x.

$$x + \frac{1}{3} = 2\left(x - \frac{2}{3}\right) - 6x$$

$$x + \frac{1}{3} = 2x - \frac{4}{3} - 6x$$

multiply by 3
everywhere

$$3x + 1 = 6x - 4 - 18x$$

$$3x - 6x + 18x = -4 - 1$$

$$15x = -5$$

$$x = \frac{-5}{15} = \boxed{-\frac{1}{3}}$$

BONUS (3 marks)Given $f(x) = x^2 + x - 1$, determine whether $f(x) = f(x - 1)$. Show your work.

$$\begin{aligned} f(x-1) &= (x-1)^2 + (x-1) - 1 \\ &= x^2 - 2x + 1 + x - 1 - 1 \\ &= x^2 - x - 1 \end{aligned}$$

BUT $f(x) = x^2 + x - 1$

so $f(x) \neq f(x-1)$