SOLUTIONS TEST 2
201-914-DW
BUSINESS MATH

1 (a) $C(10) = 300(10) + 0.1(10)^2 + 1200$
$= 4210$

THE COST OF Producing 10 UNITS is \$4210

(b) $C(100)$ is the cost OF producing 100 UNITS

(c) $C(100) = 300(100) + 0.1(100)^2 + 1200$
$= 32200$

THE COST is \$32200 to produce 100 UNITS.

2 (a) $f(-1) = (-1-1)^2$
$= (-2)^2 = 4$

(b) $g(-2) = 2(-2)+1$
$= -3$

(c) $(f \circ f)(-1)$
$= f(f(-1))$
$= f(4) = (4-1)^2 = 3^2 = 9$

(d) $(f \circ g)(x) = f(g(x))$
$= f(2x+1) = (2x+1-1)^2$
$= (2x)^2 = 4x^2$

(e) $(g \circ f)(x) = g((x-1)^2)$
$= 2(x-1)^2 + 1$
$= 2(x^2-2x+1)+1 = 2x^2-4x+3$

3 $q^2+8q+20 = 100-4q-q^2$

$2q^2+12q-80=0$

$q^2+6q-40=0$

$(q+10)(q-4)=0$

$q=-10 \quad q=4$

Negative quantities impossible

so $q=4$

$p = q^2+8q+20$

$p = 4^2+8\cdot 4+20$

$= 68$

Equilibrium point is $(4, 68)$.

4 (a) vertex $x = \frac{-b}{2a} = \frac{-80}{2(-0.4)} = 100$ units

(b) $p = 80(100) - 0.4(100)^2 - 200$

$= 3800$

$(100, 3800)$

3800 \$

(c) y-intercept $x=0$

$p = 80(0) - 0.4(0)^2 - 200$

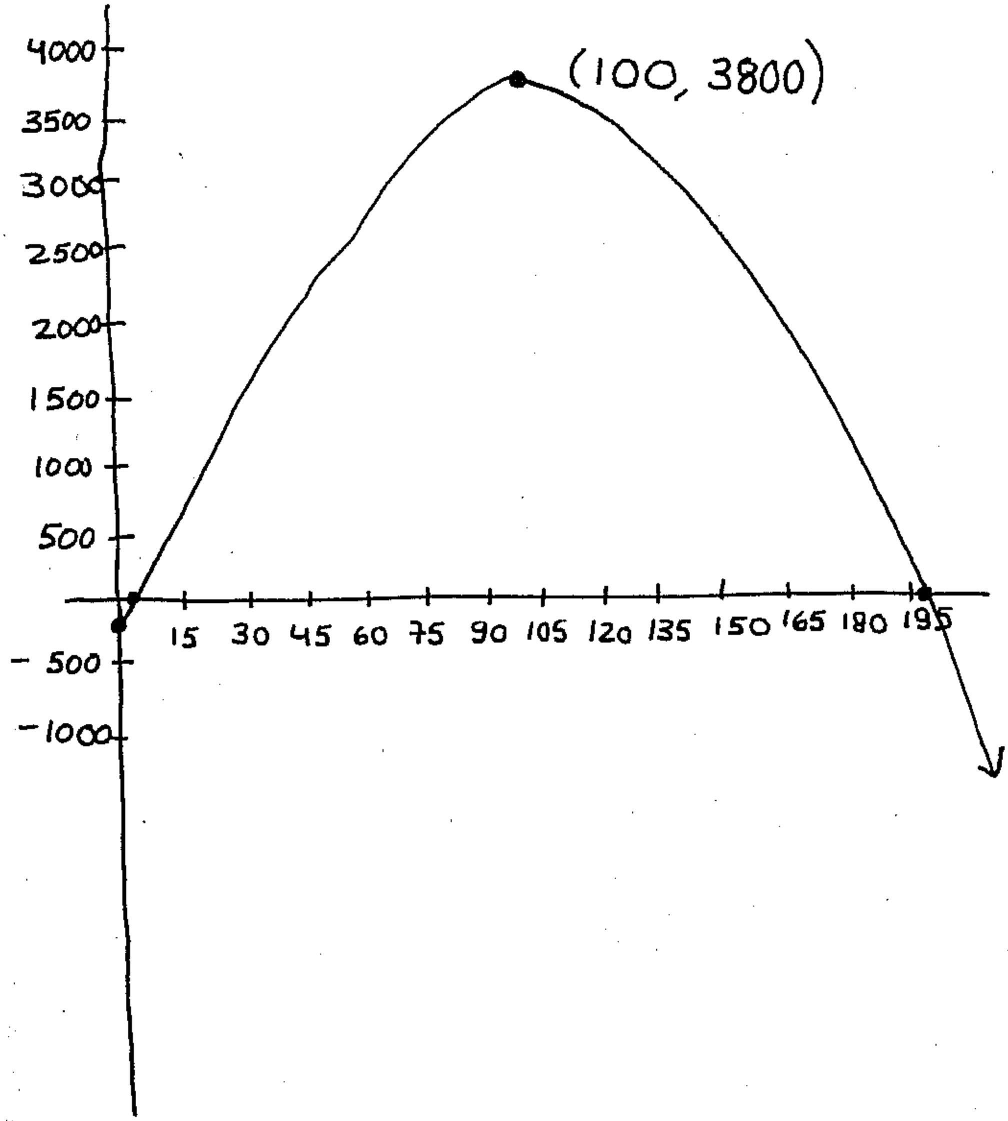
$(0, -200)$

x-intercepts $y=0$

$0 = 80x - 0.4x^2 - 200$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-80 \pm \sqrt{80^2-4(-0.4)(-200)}}{2(-0.4)}$$

$$= -\frac{80 \pm 78}{-0.8} = 2.5 \text{ or } 197.5$$



5. demand ; Two points on the line

$(45,10)$; $(20,60)$

$$m = \frac{60-10}{20-45} = \frac{50}{-25} = -2$$

$$p = -2q + b$$
$$10 = -2(45) + b$$
$$b = 100$$

$p = -2q + 100$ demand

supply ; Two points on the line

$(35,30)$, $(70,50)$

$$m = \frac{50-30}{70-35} = \frac{20}{35} = 4/7$$

$$p = 4/7\,q + b$$
$$30 = 4/7(35) + b \implies b = 10$$

$p = 4/7\,q + 10$ supply

Equilibrium when supply = demand

$$-2q + 100 = \frac{4}{7}q + 10$$
$$90 = \frac{18}{7}q$$
$$q = 35$$

so equilibrium is $q = 35$, $p = 30$.

6 (a) Revenue $R(x) = 50x$
Cost $C(x) = 30x + 10000$

(b)
$$R(x) = C(x)$$
$$50x = 30x + 10000$$
$$20x = 10000$$
$$x = 500$$

500 watches must be sold to break even

7 (a) cost $C(x) = \left(\frac{1}{4}x + 1250\right)x + 250$
$$C(x) = \frac{1}{4}x^2 + 1250x + 250$$

Revenue $R(x) = \left(1400 - \frac{2}{3}x\right)x$
$$= 1400x - \frac{2}{3}x^2$$

(b) $C(x) = R(x)$
$$\frac{1}{4}x^2 + 1250x + 250 = 1400x - \frac{2}{3}x^2$$
$$\frac{11}{12}x^2 - 150x + 250 = 0$$

$$x = \frac{150 \pm \sqrt{(150)^2 - 4\left(\frac{11}{12}\right)(250)}}{2\left(\frac{11}{12}\right)}$$

$$= \frac{150 \pm 146.91}{1.83333}$$

$x = 161.95$ or 1.685

(c) $R(x) = 1400x - \frac{2}{3}x^2$

$$x = \frac{-b}{2a} = \frac{-1400}{2\left(\frac{-2}{3}\right)} = 1050$$

$$R(1050) = 1400(1050) - \frac{2}{3}(1050)^2$$
$$= 735000$$

(d) $P(x) = R(x) - C(x)$

$$= \left(1400x - \frac{2}{3}x^2\right) - \left(\frac{1}{4}x^2 + 1250x + 250\right)$$

$$= -\frac{11}{12}x^2 + 150x - 250$$

(e) MAX profit AT vertex

$$x = \frac{-b}{2a} = \frac{-150}{2(-11/12)} = 81.81 \text{ Ton}$$

Price per unit

$$1400 - \frac{2}{3}x$$

$$= 1400 - \frac{2}{3}(81.81)$$

$$= 1345.45$$

When the price is 1345.45 \$ / Ton the profit is maximised.

8 (a) $1.3 = \log_2 x$

$2^{1.3} = 2^{\log_2 x}$

$x = 2.46$

(b) $\log_5(x-1) = 1.2$

$5^{\log_5(x-1)} = 5^{1.2}$

$x-1 = 5^{1.2}$

$x = 5^{1.2}+1$

$x = 7.90$

(c) $3e^x = 2.2$

$\ln(3e^x) = \ln(2.2)$

$\ln 3 + \ln e^x = \ln 2.2$

$\ln 3 + x = \ln 2.2$

$x = -0.31$

(d) $3 \cdot 2^x = 4^{2x}$

$\ln(3 \cdot 2^x) = \ln(4^{2x})$

$\ln 3 + \ln 2^x = \ln 4^{2x}$

$\ln 3 + x\ln 2 = 2x \ln 4$

$1.099 + 0.693x = 2.77x$

$1.099 = 2.0796x$

$x = 1.89$

9 (a) $S = 2000 \cdot (2^{-0.1(10)})$
$= 1000$

(b) $500 = 2000 \cdot 2^{-0.1x}$
$0.25 = 2^{-0.1x}$
$\ln 0.25 = \ln 2^{-0.1x}$
$\ln 0.25 = -0.1x \ln 2$
$-0.1x = \dfrac{\ln 0.25}{\ln 2}$

$x = 20$

At 20 days the sales will be 500\$; so they should start then.

10 (a) $P = \dfrac{300}{\ln(100010)} = \$26.06$

(b) $R = (26.06)(100\,000)$
$= 2\,606\,000$ \$

(c) $P = \dfrac{300}{\ln(x+10)}$

$110.78 = \dfrac{300}{\ln(x+10)}$

$\ln(x+10) = \dfrac{300}{110.78}$

$x+10 = e^{300/110.78}$

$x = 15$

(BONUS)

$$\log_a x + 3\log_a y - \frac{\log_a z}{\log_a 3}$$

$$= \log_a x + \log_a y^3 - \left(\frac{1}{\log_a 3}\right)\log_a z$$

$$= \log_a (xy^3) - \log_a z^{(1/\log_a 3)}$$

$$= \log_a \left[\frac{xy^3}{z^{1/\log_a 3}}\right]$$

MY MOVIE CHOICE : SPACE BALLS