

TEST 3 – Business Mathematics
(201-914-DW 03)
Tuesday December 1st 2009
Instructor: Emilie Richer

Name: SOLUTIONS

INSTRUCTIONS

- The test is marked out of 50 marks
- You have 1 hour and 45 minutes to complete the test
- A scientific calculator is permitted

SHOW ALL YOUR WORK

This means:

- Indicate which formulas you are using
- Don't simply put an answer down (unless it comes directly from your calculator)

FORMULAS

Simple Interest

$$I = Prt$$

$$S = P + I$$

For all formulas:

i = interest rate per period

Compound Interest

$$S = (1 + i)^n$$

$$APY = (1 + i)^m - 1$$

Continuous Compounding

$$S = Pe^{rt}$$

$$APY = e^r - 1$$

Ordinary Annuities

$$S = \frac{R \left((1 + i)^n - 1 \right)}{i}$$

$$A_n = \frac{R \left(1 - (1 + i)^{-n} \right)}{i}$$

Annuities Due

$$S_{\text{due}} = \frac{R \left((1 + i)^n - 1 \right)}{i} (1 + i)$$

$$A_{n,\text{due}} = \frac{R \left(1 - (1 + i)^{-n} \right)}{i} (1 + i)$$

Deferred Annuity

$$A_{n,k} = \frac{R \left(1 - (1 + i)^{-n} \right)}{i} (1 + i)^{-k}$$

Question 1 (5 marks)

A couple inherits \$89 000. They would like to invest the money and withdraw 500\$ at the end of every month until there is 0\$ left of the inheritance. If money can be invested at 3.5% compounded monthly how long will the inheritance last?

$$A_n = 89000$$

$$R = 500$$

$$i = \frac{0.035}{12}$$

$$n = ?$$

ORDINARY ANNUITY (END OF MONTH)

$$A_n = \frac{R (1 - (1+i)^{-n})}{i}$$

$$89000 \left(\frac{0.035}{12} \right) = 500 \left(1 - \left(1 + \frac{0.035}{12} \right)^{-n} \right)$$

$$-0.48083 = - \left(1 + \frac{0.035}{12} \right)^{-n}$$

$$\ln(0.48083) = -n \ln \left(1 + \frac{0.035}{12} \right)$$

$$n = 251.42$$

IT WILL LAST 251 MONTHS

OR 20 YEARS & 11 MONTHS

Question 2 (5 marks)

Find the APY for the following three investments:

- (a) 5% compounded weekly
- (b) 5% compounded monthly
- (c) 5% compounded yearly
- (d) 5% compounded continuously

$$(a) \quad i = \frac{0.05}{52}$$

$$APY = \left(1 + \frac{0.05}{52} \right)^{52} - 1 = \boxed{0.05125} \quad \text{OR} \quad \boxed{5.125\%}$$

$$(b) \quad APY = \left(1 + \frac{0.05}{12} \right)^{12} - 1 = \boxed{0.05116} \quad \text{OR} \quad \boxed{5.116\%}$$

$$(c) \quad APY = (1 + 0.05)^1 - 1 = \boxed{0.05} \quad \text{OR} \quad \boxed{5\%}$$

$$(d) \quad APY = e^{0.05} - 1 = \boxed{0.05127} \quad \text{OR} \quad \boxed{5.127\%}$$

Question 3 (5 marks)

A small business that manufactures miniature wax models of celebrities decides to invest \$1000 at the end of each month in order to purchase a machine that makes perfect wax replicas of Brad Pitt (their top selling wax model). This machine costs a whopping \$130 000. They can invest money at 5.2% compounded monthly, how long will it take them to accumulate enough money for their Brad Pitt making machine.

ORDINARY ANNUITY

$$S = 130\,000$$

$$i = \frac{0.052}{12}$$

$$R = 1000$$

$$n = ?$$

$$S = \frac{R((1+i)^n - 1)}{i}$$

$$130\,000 = \frac{1000 \left(\left(1 + \frac{0.052}{12}\right)^n - 1 \right)}{\frac{0.052}{12}}$$

$$130\,000 \left(\frac{0.052}{12} \right) = 1000 \left(\left(1 + \frac{0.052}{12}\right)^n - 1 \right)$$

$$1.5633 = \left(1 + \frac{0.052}{12}\right)^n$$

$$\ln(1.5633) = n \ln(1.004333)$$

$$\text{It will take } \boxed{104 \text{ MONTHS}} \quad n = 103.34$$

$$\text{OR } \boxed{8 \text{ YEAR } \& \text{ 8 MONTHS}}$$

Question 4 (5 marks)

Morgan wants to invest money into a college fund for her young son. She plans on depositing 425\$ at the beginning of every quarter for the next 18 years. How much will her investment be worth after these 18 years if it earns 3.8% compounded quarterly?

Annuities due

$$n = 18 \cdot 4 = 72$$

$$i = \frac{0.038}{4} = 0.0095$$

$$R = 425$$

$$S = ?$$

$$S = \frac{R((1+i)^n - 1)(1+i)}{i}$$

$$= \frac{425 \left((1.0095)^{72} - 1 \right) (1.0095)}{0.0095}$$

$$= \boxed{\$44\,050.91}$$

Question 5 (10 marks)

Yann doesn't trust any bank with his money, so he keeps it under his bed in a shoebox. He has been placing his weekly earnings of \$625 in his shoebox at end of each week for the past 16 years.

- How much does he have in his shoebox?
- If Yann changed his mind and invested all of the shoebox's contents into the CRA-P Bank at 1.3% interest, how much interest could he earn in one year?
- Suppose that instead of depositing his money in a shoebox for 16 years, Yann had invested his weekly earnings at the end of each week in the NT-REST Bank at 1.3% interest compounded weekly, how much would he have in his shoebox?

(a) $52 \text{ weeks} \cdot 625 \cdot 16 \text{ years}$

$$= 520000$$

He HAS \$520 000 in his shoe box

(b) Simple interest

$$I = Prt$$
$$= (520000)(0.013)(1)$$

$$= 6760$$

He could EARN \$6760 in one year.

(c) Ordinary Annuity

$$S = ?$$

$$R = 625$$

$$n = 52 \cdot 16$$
$$= 832$$

$$i = \frac{0.013}{52} = 0.00025$$

$$S = \frac{R((1+i)^n - 1)}{i}$$

$$= \frac{625((1.00025)^{832} - 1)}{0.00025}$$

$$= 577\,952.91$$

He would have \$577 952.91

Question 6 (5 marks)

A company wants to have \$ 250 000 after 3.5 years to modernize its production equipment. How much money should the company deposit at the beginning of each quarter into an account that earns 6.3% compounded quarterly if they want to achieve this goal?

Annuities due

$$i = \frac{0.063}{4} = 0.01575$$

$$n = (3.5) \cdot 4 = 14$$

$$S = 250\,000$$

$$R = ?$$

$$S = \frac{R \left((1+i)^n - 1 \right) (1+i)}{i}$$

$$250\,000 = \frac{R \left((1.01575)^{14} - 1 \right) (1.01575)}{0.01575}$$

$$R = 15\,850.73$$

The deposits should be of \$ 15850.73

Question 7 (5 marks)

On his 48th birthday, a man decides that he wants to set aside enough money to provide an income of \$300 at the end of each month from his 60th to his 65th birthday. If he earns 6% compounded monthly, how much will this supplemental retirement plan cost him on his 48th birthday?

DEFERRED ANNUITY

$$K = 12 \text{ YEARS}$$

$$= 12 \times 12 = 144 \text{ MONTHS}$$

$$n = 5 \cdot 12 = 60$$

$$R = 300$$

$$i = \frac{0.06}{12} = 0.005$$

$$A_{nK} = \frac{R \left(1 - (1+i)^{-n} \right) (1+i)^{-K}}{i}$$

$$= \frac{300 \left(1 - (1.005)^{-60} \right) (1.005)^{-144}}{0.005}$$

$$= 7566.82$$

He must invest \$ 7566.82 on his 48th birthday

Question 8 (10 marks)

Charlie has just won the "Sucker-Born-Every-Minute" Lottery! For his prize, he has the choice between receiving \$10 000 at the end of each month for the next 40 years OR a lump sum prize of \$1.5 million (\$1 500 000). Money can earn 7.2% compounded monthly.

- (a) Which prize would the *lottery company* want Charlie to choose? How much money will they save if Charlie makes this decision?
 (b) Which prize should Charlie choose? How much more money will he make with this choice?

(a) They MUST
 PAY \$1 500 000 NOW

OR $A_n = ?$
 $i = \frac{0.072}{12}$
 $n = 12 \cdot 40$
 $= 480$
 $R = 10\,000$

$$A_n = \frac{10000 [1 - (1.006)^{-480}]}{0.006}$$

$$= 1\,572\,300.13$$

They would thus PREFER
 that CHARLIE choose the
lump sum. This way they
 would save \$72 300.13

(b) CHARLIE
 CAN EITHER
 TAKE 1.5 million
 NOW & invest it
 For 40 years:

$S = ?$
 $n = 40 \cdot 12 = 480$
 $i = 0.006$
 $P = 1\,500\,000$

$$S = P(1.006)^{480}$$

$$= (1\,500\,000)(1.006)^{480}$$

$$= 26\,492\,442.76$$

OR Receive \$10 000
 payments

$S = ?$
 $n = 480$
 $R = 10\,000$
 $i = 0.006$

$$S = \frac{R((1+i)^n - 1)}{i}$$

$$= \frac{10000((1.006)^{480} - 1)}{0.006}$$

$$= 27\,769\,380.84$$

CHARLIE should choose
 the installments. He can earn
\$1 276 938.09 more money.